Section 6.6 #15

A leaky 10-kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12-m level. How much work is done to lift the bucket?

Solution

Bucket: The force of the bucket is 10 kg * 9.8 (gravity constant) and the distance the bucket is lifted is 12 m, so the work involved in lifting the empty bucket is 9.8(10)*12 = 1176 *J (*joules)

Rope: At a height of *y* meters ($0 \le y \le 12$), the mass of the rope is 0.8 kg/m (12 – *y*) m and the distance lifted is Δy , so the work to lift the rope is $\int_0^{12} 9.8 * 0.8(12 - y)\, dy = 564.48\ J$

Water: The mass of the water at height *y* meters is $\frac{36}{12} kg/m(12 - x)m = 36 - 3x\ kg$ and the distance the water is lifted is Δy , so the work to lift the water is $\int_0^{12} 9.8(36 - 3y)dy = 2116.8\ J$.

Adding all the work done to lift the bucket, rope and water you set about
1176 (bucket) + 564.48 (rope) + 2116.8 (water) =3857 *J*

When dealing with the water, you don't multiply by 1000 kg/m^3 because you weren't given the volume of the water, you were given the mass. You do multiply mass by 9.8 (the gravity constant) to get the force because F = ma [Force = mass x acceleration (gravity)]