1. State whether each of the following is True or False. If False, give a brief explanation, graph, or counterexample, or provide the correct statement.

F a.) The average rate of change of a function \( Q \) with respect to \( t \) over an interval can be symbolically represented by \( \frac{\Delta Q}{\Delta t} \). Should be \( \frac{\Delta Q}{\Delta t} \).

F b.) If a line has slope 2 and \( y \)-intercept -3, then its equation may be written \( y = 3x + 2 \). Should be \( y = -3 + 2x \).

T c.) The process of estimating a value within the range for which we have collected data is called interpolation.

F d.) Extrapolation tends to be more reliable than interpolation. It is less reliable.

F e.) The domain of \( f(x) = \frac{4}{x - 3} \) consists of all real numbers \( x, x \neq 0 \).

Domain is all \( x, x \neq 3 \).

F f.) The range of \( f(x) = \frac{1}{x} \) is all real numbers. Range is all \( y, y \neq 0 \).

F g.) The units of output of a function are the same as the units of input. Consider the formula for the area of a circle, \( A = \pi r^2 \). If the units of input, radius, is given in feet, the units of output are in square feet.

F h.) If a function is concave up, it must be increasing. Pictured below is the graph of a concave up, decreasing function.

F i) An increasing linear function has an increasing rate of change. All linear functions have a constant rate of change.

2. Consider the information in the following table:

<table>
<thead>
<tr>
<th>( s )</th>
<th>3</th>
<th>0</th>
<th>-1</th>
<th>6</th>
<th>2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>-2</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

a.) Could \( t \) be a function of \( s \)? Explain. Yes. For each input \( s \), there is exactly one corresponding output, \( t \).

b.) Could \( s \) be a function of \( t \)? Explain. No. For the input \( t=5 \), there are 2 different outputs, \( s \).
3. Match the $r$ values with the scatter plots shown.

\[ r = 0.9 \quad d \quad r = 0.5 \quad c \quad r = 0 \quad e \]

\[ r = -0.5 \quad b \quad r = -0.9 \quad a \]

4. Find the equation of the line that contains the point $(-3, 5)$ and is perpendicular to the line $y = 2 - \frac{3}{7} x$.

Answer: $y = 12 + \frac{7}{3} x$

5. a.) Find the domain of the function $h(x) = \frac{2}{\sqrt{7-x}}$. Domain: All $x < 7$. (Note that the expression $7 - x > 0$).

b.) Find the range of the function $s(x) = \frac{6 - x}{3 + x}$. Solve for $x$: $x = \frac{6 - 3y}{1 + y}$. So the range is all $y \neq -1$

6. An airplane has room for 300 coach-fare seats. It can replace any 3 coach seats with 2 first-class seats. Suppose the airplane is configured with $x$ coach seats and $y$ first-class seats. (Assume no space is wasted.)

a.) If $y = 0$, what is $x$? 300
b.) If $x = 0$, what is $y$? 200
c.) If $y$ is a linear function of $x$, sketch a graph of $y$ versus $x$. Label both the $x$- and $y$-intercepts.

d.) Find a formula for $y$ in terms of $x$. Answer: $y = 200 - \frac{2}{3} x$

e.) Explain the meaning of the slope and the intercepts of the formula you found in part (d) in terms of the airplane’s seat configuration. Slope: For every 3 coach seats we add, we must remove 2 first class seats. Y-intercept: The maximum number of first class seats (if there are no coach seats) is 200. X-intercept: The maximum number of coach seats (if there are no first class seats) is 300.

f.) State a reasonable domain and range for the function found in part (d). Domain: $0 \leq x \leq 300$.
Range: $0 \leq y \leq 200$
7. Suppose that \( f(x) \) is invertible and that both \( f \) and \( f^{-1} \) are defined for all values of \( x \). Let \( f(2) = 3 \) and \( f^{-1}(5) = 4 \). Evaluate the following expressions, or, if the given information is insufficient, write unknown.

a.) \( f^{-1}(3) \) __2____  

b.) \( f^{-1}(4) \) unknown  

c.) \( f(4) \) __5______

8. Values of \( f \) and \( g \) are given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

a) Evaluate \( f(1) \) __2____ and \( g(3) \) __4_____.  

b.) Identify which of the functions could represent a linear function, and find a formula for it.  

\( f \) is linear. The formula is \( f(x) = -1 + 3x \)

9. If \( q(x) = x^2 - 3x \), evaluate and simplify:

a.) \( q(-2) = 10 \)  
b.) \( q(a + 1) = a^2 - a - 2 \)

c.) Find the average rate of change of \( q \) over the interval \([-2, 1]\).

\[ \Delta q = \frac{q(1) - q(-2)}{1 - (-2)} = \frac{-2 - 10}{3} = \frac{-12}{3} = -4 \]

d.) Based on your answer in part (c), is \( q \) an increasing or decreasing function on the interval \([-2, 1]\)?  

Decreasing, because the average rate of change on this interval is negative.

10. Write a formula for the function \( y = f(x) \) pictured below. Answer: \( f(x) = \begin{cases} 1, & x < 0 \\ x^2, & 0 \leq x < 2 \\ x, & x \geq 2 \end{cases} \)

11. Let \( w(m) \) give the weight (in pounds) of an average-sized baby girl who is \( m \) months old.

a.) An average six-month old girl weighs 15 pounds. Express this using the function \( w(m) \). \( w(6) = 15 \)

b.) In practical terms, what does the statement \( w(12) = 21 \) tell you about baby girls? The average 12-month old baby girl weighs 21 pounds.

c.) In practical terms, what does the statement \( w(p) = 14 \) tell you about \( p \). \( p \) is the age in months when an average baby girl weighs 14 pounds.

d.) An eight-month old baby girl weighs \( w(13) \) pounds. Is she of average weight, above average weight, or below average weight? Explain. Above average; she weighs what an average 13-month old girl weighs.
12. A T-shirt company charges a set-up fee of $40 for each order, plus $10 per shirt for the first 20 shirts, and then $8 per shirt for every shirt after the first twenty.

a.) Find the cost of ordering 20 shirts. $$\text{\$240}$$

b.) Find the cost of ordering 50 shirts. $$\text{\$480}$$

c.) Sketch a graph of the cost function $$C(n)$$, where $$C$$ is the cost of an order of $$n$$ shirts. (Be sure to label key points.) See below.

c.) Find a formula for $$C(n)$$. Ans: $$C(x) = \begin{cases} 40 + 10x, & 0 < x \leq 20 \\ 240 + 8(x - 20), & x > 20 \end{cases}$$

d.) A rival company advertises that they keep things simple by not charging a set-up fee, and they only charge $10 per shirt. How large (or small) should your order be to make this second company a better deal? Figure out where the graphs of the functions intersect. The second company is a better deal when ordering between 0 and 40 shirts (the cost is the same for both companies for an order of 40 shirts). See graph above.

13. Find a formula and sketch a quadratic function that has all of the following properties: concave-down, $$y$$-intercept at 5, zeros at $$x = -3$$ and $$x = 1$$. Ans: $$f(x) = a(x + 3)(x - 1)$$. Use the point $$(0, 5)$$ to find the value of $$a$$. $$a = -\frac{5}{3}$$, so $$f(x) = -\frac{5}{3}(x + 3)(x - 1)$$. 
14. Calculate successive rates of change for the function \( g(t) \) and tell whether you expect the graph of \( g \) to be concave up or concave down.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(t) )</td>
<td>-2</td>
<td>0</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

On the interval \((0, 2)\): \( \frac{\Delta g}{\Delta t} = \frac{2}{2} = 1 \), on the interval \((2, 4)\): \( \frac{\Delta g}{\Delta t} = \frac{4}{2} = 2 \).

On the interval \((4, 6)\): \( \frac{\Delta g}{\Delta t} = \frac{12}{2} = 6 \). The graph of \( g \) will be concave up on the interval \((0, 6)\), because successive average rates of change are increasing.

15. Given the graph of the function \( y = p(x) \):

a.) Evaluate \( p(a) \) = i and \( p(0) \) = h.

b.) Is \( p\left(\frac{1}{2}d\right) \) closer to \( g \), \( h \), or \( i \)? Closer to \( h \). Use the graph to estimate the output corresponding to the input halfway between 0 and \( d \).

c.) Is \( p(-b) \) positive, negative, or equal to zero? Negative. Note that since \( b < 0 \), then \( -b > 0 \) and \( 0 < -b < d \), which means \( p(-b) \) will be negative.

d.) If \( h = p(z) \) and \( z = p(x) \), what is \( x \)? Answer: Since \( h = p(z) \) implies that \( z = 0 \), we now have \( z = p(x) \), which is the same as \( 0 = p(x) \), which means that \( x = b \).