MATH 2412 – Precalculus – Review for Exam 1

Work these on separate paper and do not write on this sheet. You must show your work.

Each problem represents a concept to review

The table gives the populations of two cities (in thousands) over a 17-year period. Use the table for problems 1 – 3.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>42</td>
<td>46</td>
<td>50</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>$P_2$</td>
<td>82</td>
<td>80</td>
<td>76</td>
<td>69</td>
<td>58</td>
</tr>
</tbody>
</table>

1) Find the average rate of change of each population on the following intervals:
   a) 1990 to 2000        b) 1995 to 2007

2) Is $P_1$ increasing or decreasing? And is the rate of change increasing, decreasing or constant? What does this say about the concavity of the graph of $P_1$?

3) Is $P_2$ increasing or decreasing? And is the rate of change increasing, decreasing or constant? What does this say about the concavity of the graph of $P_2$?

4) Find the formula for the linear function where $g(100) = 2000$ and $g(400) = 3800$

5) Find the formula for the linear function $P = h(t)$ which gives the size of a population that begins with 12,000 members and grows by 225 members each year.

6) Give the linear equation that models the data in the table.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>0.736</td>
<td>0.614</td>
<td>0.492</td>
<td>0.37</td>
</tr>
</tbody>
</table>

7) The table gives the daily low temperature for a week in NYC during July.

<table>
<thead>
<tr>
<th>Date, $d$</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp, $T$</td>
<td>73</td>
<td>77</td>
<td>69</td>
<td>73</td>
<td>75</td>
<td>75</td>
<td>70</td>
</tr>
</tbody>
</table>

   a) Is the temperature, $T$, a function of the date, $d$? Explain.
   b) Is the date, $d$, a function of the temperature, $T$? Explain.

8) For $f(x) = x^2 + 1$

   a) Find the average rate of change between $(-1, f(-1))$ and $(3, f(3))$
   b) Find the average rate of change between $(x, f(x))$ and $(x + h, f(x + h))$

9) The population $P(t)$, in millions, of a country in year $t$ is given by $P(t) = 22 + 0.3t$

   a) What is the country’s initial population?
   b) What is the average rate of change of the population, in millions of people per year?

10) A woodworker sells rocking horses. The start-up costs, including tools, plans and advertising, total $5000. Labor and materials for each horse cost $350.

   a) Find a formula for total cost, $C$, in terms of the number of rocking horses carved, $n$.
   b) What is the rate of change of the function $C$?

11) Find the equation of the linear function $g$ whose graph is perpendicular to the line $5x - 3y = 6$. The two lines intersect at $x = 15$

12) Find the formula for the linear function $h(x)$ whose graph intersects the graph of $k(x) = 30(0.2)^x$ at $x = -2$ and $x = 1$
13) The graph shows the function \( f(x) = 12 - 0.5(x + 4)^2 \) and the linear function \( g \). Find a formula for \( g \).

14) Find the coordinates of the point \( P \) in the figure.

15) A theater manager graphed weekly profits as a function of the number of patrons and found that the relationship was linear. One week, the profit was \$11,328 \) when 1324 patrons attended. Another week 1529 patrons produced a profit of \$13,275.50 \)

a) Find a formula for weekly profit, \( y \), as a function of the number of patrons, \( x \).

b) Interpret the slope and \( y \)-intercept within the context.

c) What is the break-even point (zero profit)?

d) If the weekly profit was \$17,759.50, how many patrons attended the theater?

16) You want to choose one long-distance telephone company from the following options:

- Company P charges \$0.37 per minute
- Company R charges \$13.95 a month plus \$0.22 per minute
- Company S charges a fixed rate of \$50 per month

Let \( P, R, \) and \( S \) represent the monthly charges using each company respectively. Let \( x \) be the number of minutes of long-distance calls per month.

a) Find formulas for \( P, R \) and \( S \) as functions of \( x \).

b) The figure gives the graphs of the functions. Which function corresponds to which graph?

c) When is Company S cheapest?

17) Solve \( f(x) = 6 \) for \( f(x) = \sqrt{20 + 2x^2} \)

Find the domain of the functions in problems 18 – 20.

18) \( f(x) = \sqrt{x^2 - 9} \)

19) \( g(x) = \frac{x + 1}{\sqrt{9 - x}} \)

20) \( h(x) = \frac{2x + 3}{5 - 4x} \)

Use \( f(x) = 1 - x \) to find the following for problems 21 – 24.

21) \( 2f(x) \)

22) \( f(x) + 1 \)

23) \( f(1 - x) \)

24) \( \left(f(x)\right)^2 \)

Use \( f(x) = x^2 + 1 \) and \( g(x) = 2x + 3 \) to find the following for problems 25 – 29.

25) \( f\left(g(1)\right) \)

26) \( g\left(f(-2)\right) \)

27) \( f\left(g(x)\right) \)

28) \( g\left(f(x)\right) \)

29) \( f\left(f(x)\right) \) (simplify)

For problems 30 – 32, let \( P = f(t) \) be the population, in millions, of a country at time \( t \) years and let \( E = g(P) \) be the daily electricity consumption, in megawatts, when the population is \( P \). Give the meaning within the context and the units of each function.

30) \( g(f(t)) \)

31) \( f^{-1}(P) \)

32) \( g^{-1}(E) \)
33) Find \( f^{-1} \) for \( f(x) = \frac{3x}{x + 4} \)

34) For \( f(x) = 12 - \sqrt{x} \), evaluate \( f(16) \) and \( f^{-1}(3) \)

Graph the function for problems 35 and 36.

35) \( f(x) = \begin{cases} 
   x^2 & \text{for } x \leq 1 \\
   2 - x & \text{for } x > 1
\end{cases} \)

36) \( f(x) = \begin{cases} 
   x + 1 & \text{for } -2 \leq x < 0 \\
   x - 1 & \text{for } 0 \leq x < 2 \\
   x - 3 & \text{for } 2 \leq x < 4
\end{cases} \)

Write the formula for the piecewise-defined function shown in the graph for problems 37 and 38.

37)

\[
\begin{array}{c|c}
\hline
x & y \\
\hline
2 & 1 \\
3 & 2.5 \\
4 & 3.5 \\
5 & 7 \\
\hline
\end{array}
\]

38)

\[
\begin{array}{c|c}
\hline
x & y \\
\hline
2 & 1 \\
3 & 2 \\
4 & 3 \\
5 & 5 \\
\hline
\end{array}
\]

39) Use the graph to fill in the missing values.
   a) \( f(0) = ? \)  b) \( f(?) = 0 \)  c) \( f^{-1}(0) = ? \)  d) \( f^{-1}(?) = 0 \)

40) Let \( j(x) = h^{-1}(x) \) where \( h \) and \( j \) are defined for all values of \( x \). Let \( h(4) = 2 \) and \( j(5) = -3 \). Evaluate if possible.
   a) \( j(h(4)) \)  b) \( j(4) \)  c) \( h(j(4)) \)  d) \( j(2) \)  e) \( j^{-1}(-3) \)  f) \( h^{-1}(-3) \)

41) Find the zeros of \( f(x) = 6x^2 - 17x + 12 \)

42) Complete the square to write the function in vertex form and give the vertex. \( f(x) = 3x^2 - 6x + 5 \)

43) Complete the square to write the function in vertex form and give the vertex. \( g(x) = -3x^2 + 24x - 36 \)
For problems 44 – 47, write the formula for the quadratic function with the given characteristics.

44) Vertex at (1, -2) and y-intercept of -5
45) Vertex at (7, 3) and containing the point (3, 7)
46) The x-intercepts are -1 and 2 and the graph contains the point (-2, 16)
47) There is only one x-intercept at ½ and a y-intercept at 3.

For problems 48 and 49, write the formula of the quadratic function shown in the graph.

50) A tomato is thrown vertically into the air at time $t = 0$. Its height, $h(t)$ (in feet), above the ground at time $t$ (in seconds) is given by $h(t) = -16t^2 + 48t$
   a) Find $t$ when $h(t) = 0$. What is happening to the tomato the first time $h(t) = 0$? The second time?
   b) Evaluate and interpret $h(2)$
   c) What is the concavity of the graph of $h(t)$?