Filter Notes

You may have memorized a formula for the voltage divider - if not, it is easily derived using Ohm's law,

\[ \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2} \]

If you recall the formula for capacitive reactance, the divider formula works fine when R₂ is replaced by a capacitor.

\[ \frac{V_o}{V_i} = \frac{-j \frac{1}{2\pi fC}}{R - j \frac{1}{2\pi fC}} = \frac{1}{1 + j2\pi fRC} \]

This is the formula for the transfer function of an elementary low pass filter. Consider two frequency ranges:

1. Very low frequencies (\( f \to 0 \))

\[ \frac{V_o}{V_i} = \frac{1}{1 + 0} \]

*Flat response - no change in amplitude and no phase shift.*

2. Very high frequencies (denominator dominated by second term)

\[ \frac{V_o}{V_i} = \frac{1}{j2\pi fRC} \]

*Amplitude dropping like 1/f - phase shift of -90°*

We define a frequency midway between these two extreme ranges, \( f = f_c \). The cutoff frequency is that frequency at which

\[ \frac{V_o}{V_i} = \frac{1}{1 + j} \]

\( V_o = 0.707 \times V_i \) - *phase shift is -45°*

So \( 2\pi f_c RC = 1 \), and

\[ f_c = \frac{1}{2\pi RC} \]
Filter Design Problem

What if we had to use passive, analog filters for our low pass and high pass processing? Design elemental RC filters for low pass \((f_c = 500\text{Hz})\) and high pass \((f_c = 20\text{Hz})\). Do we want everyone to have the same solution? Most everybody has a bunch of \(0.1\ \mu\text{F}\) capacitors laying around - let's solve for the resistors that we need to go with them. What is the order of these filters? Are they Butterworth? If these analog filters were used on our two cases, how much improvement in signal/noise ratio should we expect?

Solution:

The elemental RC filters have similar forms

\[ R = \frac{1}{2\pi f C} \]

Low pass - \(f_c = 500\text{Hz}\) - \(R = 3,185\ \text{Ohms}\) (Use a 3.2K), and

High pass - \(f_c = 20\text{Hz}\) - \(R = 79,618\ \text{Ohms}\) (Use an 80K).

These are both first order filters - Response in the stop band is rolling off at -20 dB/decade or -6 dB/octave. For both filters the cutoff frequency is one octave away from the noise - therefore the noise would be smashed by a factor of 2. Expect the 0.1 volt noise amplitudes to be reduced to 0.05 volts - a 6 dB improvement in S/N ratio in both cases.

Visualize the filters by looking at Bode plots. Sketch the asymptotic filter response with frequency on a logarithmic scale and amplitude on a linear scale, but in dB. Here is a link to a bio of Professor Bode:

http://www.geocities.com/neveyaakov/electro_science/bode.html

This is how most everybody visualizes frequency responses.