CHEM 1412  
Chapter 26 Homework Answers

1. Indicate the number of protons and neutrons in the following nuclei:
(a) $^{59}_{28}$Ni  \[ p^+ = 28 \quad n = 31 \]
(b) $^{94}_{40}$Zr  \[ p^+ = 40 \quad n = 54 \]
(c) oxygen-18  \[ p^+ = 8 \quad n = 10 \]
(d) $^{107}_{47}$Ag  \[ p^+ = 47 \quad n = 60 \]
(e) $^{31}_{15}$P  \[ p^+ = 15 \quad n = 16 \]
(f) indium-115  \[ p^+ = 49 \quad n = 66 \]

2. What is mass deficiency? What is nuclear binding energy? How are the two related?
Mass deficiency ($\Delta m$) is the difference between the calculated mass of an atom (sum of rest masses for $p^+ + e^- + n$) and the actual mass of an atom.

Nuclear binding energy ($\Delta E$) is the energy that holds the nucleus together. It is the mass deficiency, converted to energy.

Relationship: $\Delta E = (\Delta m)c^2$

3. The actual mass of $^{62}$Ni is 61.9283 amu. What is the nuclear binding energy in kJ/mol for this isotope?

\[
\text{mass (calc.)} = 28(0.00054858 \text{ amu}) = 0.015360 \text{ amu} \\
28(1.00728 \text{ amu}) = 28.2038 \text{ amu} \\
34(1.00867 \text{ amu}) = 34.2948 \text{ amu} \\
62.5140 \text{ amu} = \text{sum of constituent particles}
\]

$\Delta m = 62.5140 \text{ amu} - 61.9283 \text{ amu} = 0.5857 \text{ amu} = 0.5857 \text{ g/mol}$

$\Delta m = \frac{0.5857 \text{ g}}{1 \text{ mol}} \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) = 5.871 \times 10^{-4} \text{ kg/mol}$

$\Delta E = (\Delta m)c^2$

$\Delta E = (5.857 \times 10^{-4} \text{ kg/mol})(3.00 \times 10^8 \text{ m/s})^2$

$\Delta E = 5.27 \times 10^{13} \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{mol} \quad \text{where} \ 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$

$\Delta E = 5.27 \times 10^{15} \text{ J/mol}$

$\boxed{\Delta E = 5.27 \times 10^{10} \text{ kJ/mol}}$
4. The actual mass of $^{108}_{46}$Pd is 107.90389 amu. What is the nuclear binding energy in kJ/mol for this nuclide?

\[
\text{mass (calc.)} = 46(0.00054858 \text{ amu}) = 0.025235 \text{ amu} \\
46(1.00728 \text{ amu}) = 46.3349 \text{ amu} \\
62(1.00867 \text{ amu}) = 62.5375 \text{ amu} \\
108.8976 \text{ amu} = \text{sum of constituent particles}
\]

\[\Delta m = 108.8976 \text{ amu} - 107.90389 \text{ amu} = 0.9937 \text{ amu} = 0.9937 \text{ g/mol}\]

\[\Delta m = 9.937 \times 10^{-4} \text{ kg/mol}\]

\[\Delta E = (\Delta m)c^2\]

\[\Delta E = (9.937 \times 10^{-4} \text{ kg/mol})(3.00 \times 10^8 \text{ m/s})^2\]

\[\Delta E = 8.94 \times 10^{13} \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{mol}\]

\[\Delta E = 8.94 \times 10^{13} \text{ J/mol}\]

\[\underline{\Delta E = 8.94 \times 10^{10} \text{ kJ/mol}}\]

5. The band of stability tells us that the neutron-to-proton ratio for small stable nuclei is 1:1. Determine the n/p$^+$ ratio for the following small nuclei and predict whether the nuclide is stable or radioactive.

(a) $^{17}$N  \quad n/p$^+ = 10/7 = 1.4$  \quad \text{particle is radioactive}

(b) $^{8}$B  \quad n/p$^+ = 3/5 = 0.6$  \quad \text{particle is radioactive}

(c) $^{18}$F  \quad n/p$^+ = 9/9 = 1.0$  \quad \text{particle is stable}

(d) $^{12}$C  \quad n/p$^+ = 6/6 = 1.0$  \quad \text{particle is stable}

(e) $^{14}$C  \quad n/p$^+ = 8/6 = 1.3$  \quad \text{particle is radioactive}

(f) $^{13}$O  \quad n/p$^+ = 5/8 = 0.62$  \quad \text{particle is radioactive}
6. Write nuclear equations for the following processes:

(a) bismuth-214 undergoes beta decay

\[ ^{214}_{83}\text{Bi} \quad \longrightarrow \quad ^{214}_{84}\text{Po} \quad + \quad ^{0}_{-1}\beta \]

(b) gold-195 undergoes electron capture

\[ ^{195}_{79}\text{Au} \quad + \quad ^{0}_{-1}\beta \quad \longrightarrow \quad ^{195}_{78}\text{Pt} \]

(c) potassium-38 undergoes positron emission

\[ ^{38}_{19}\text{K} \quad \longrightarrow \quad ^{38}_{18}\text{Ar} \quad + \quad ^{0}_{+1}\beta \]

(d) plutonium-242 emits alpha radiation

\[ ^{242}_{94}\text{Pu} \quad \longrightarrow \quad ^{238}_{92}\text{U} \quad + \quad ^{4}_{2}\alpha \]

7. Consider a radioactive nuclide with a neutron-to-proton ratio that is above the band of stability. What mode(s) of decay might be expected for this nuclide, and why?

This nucleus needs to decrease neutrons and/or increase protons. Two types of decay are plausible:

- \( \beta \) emission turns neutrons into protons:
  \[ ^{1}_{0}\text{n} \quad \longrightarrow \quad ^{1}_{1}\text{p} \quad + \quad ^{0}_{-1}\beta \]
- Neutron emission: decreases the number of neutrons

8. Answer the same questions as in Problem 7, but for a radioactive nuclide that has a neutron-to-proton ratio below the band of stability.

In this case the nuclide must increase neutrons and/or decrease protons. Three types of processes are possible:

- Positron emission turns protons into neutrons:
  \[ ^{1}_{1}\text{p} \quad \longrightarrow \quad ^{1}_{0}\text{n} \quad + \quad ^{0}_{+1}\beta \]
- Proton emission: decreases the number of protons
- K-capture: turns protons into neutrons:
  \[ ^{1}_{1}\text{p} \quad + \quad ^{0}_{-1}\beta \quad \longrightarrow \quad ^{1}_{0}\text{n} \]
9. The following nuclei are radioactive. Predict at least one mode of decay this is likely for each example.
(a) $^{20}\text{O}_{8}$ \hspace{1cm} n/p$^+ = 12/8 = 1.5$ \hspace{1cm} n/p$^+$ too high \hspace{1cm} beta or neutron emission
(b) $^{46}\text{V}_{23}$ \hspace{1cm} n/p$^+ = 23/23 = 1.0 \hspace{1cm} n/p$^+$ too low \hspace{1cm} positron or proton emission, K-capture
(c) $^{213}\text{Po}_{85}$ \hspace{1cm} nucleus too big \hspace{1cm} alpha emission
(d) $^{20}\text{Mg}_{12}$ \hspace{1cm} n/p$^+ = 8/12 = 0.67 \hspace{1cm} n/p$^+$ too low \hspace{1cm} positron or proton emission, K-capture
(e) $^{221}\text{Fr}_{87}$ \hspace{1cm} nucleus too big \hspace{1cm} alpha emission
(f) $^{45}\text{K}_{19}$ \hspace{1cm} n/p$^+ = 26/19 = 1.4 \hspace{1cm} n/p$^+$ too high \hspace{1cm} beta or neutron emission

10. Fill in the missing symbol in the following nuclear bombardment reactions.
(a) $^{23}\text{Na}_{11} + \frac{1}{1}\text{P}$ \hspace{1cm} $\rightarrow$ $^{23}\text{Mg}_{12}$ + $\frac{1}{0}\text{n}$
(b) $^{59}\text{Co}_{27} + \frac{1}{0}\text{n}$ \hspace{1cm} $\rightarrow$ $^{56}\text{Mn}_{25}$ + $\frac{\alpha}{2}$
(c) $^{96}\text{Mo}_{42} + \frac{4}{2}\alpha$ \hspace{1cm} $\rightarrow$ $^{100}\text{Tc}_{43}$ + $\frac{0}{1}\text{I}$
(d) $^{209}\text{Bi}_{83} + \frac{2}{1}\text{H}$ \hspace{1cm} $\rightarrow$ $^{210}\text{Po}_{84}$ + $\frac{1}{0}\text{n}$
(e) $^{238}\text{U}_{92} + \frac{16}{8}\text{O}$ \hspace{1cm} $\rightarrow$ $^{240}\text{Fm}_{100}$ + $5\frac{1}{0}\text{n}$
(f) $^{232}\text{Th}_{90} + \frac{12}{6}\text{C}$ \hspace{1cm} $\rightarrow$ $^{240}\text{Cm}_{96}$ + $4\frac{1}{0}\text{n}$
(g) $^{28}\text{Al}_{13} + \frac{1}{1}\text{H}$ \hspace{1cm} $\rightarrow$ $^{29}\text{Si}_{14}$ + $\frac{0}{0}\gamma$
(h) $^{25}\text{Mg}_{12} + \frac{1}{1}\text{P}$ \hspace{1cm} $\rightarrow$ $^{26}\text{Al}_{13}$ + $\frac{1}{0}\text{n}$
(i) $^{40}\text{Ar}_{18} + \frac{\alpha}{2}\text{C}$ \hspace{1cm} $\rightarrow$ $^{43}\text{K}_{19}$ + $\frac{1}{1}\text{H}$
11. Write the symbol for the daughter nuclei in the following nuclear bombardment reactions.

(a) $^{60}_{28}$Ni (n,p) $^{60}_{28}$Ni $+ \frac{1}{2}n \rightarrow \frac{1}{2}P + \frac{60}{27}$C

(b) $^{98}_{42}$Mo (n,β) $^{98}_{42}$Mo $+ \frac{1}{2}n \rightarrow \frac{1}{2}β + \frac{99}{43}$Te

(c) $^{35}_{17}$Cl (p,α) $^{35}_{17}$Cl $+ \frac{4}{2}α \rightarrow \frac{32}{16}$S

(d) $^{20}_{10}$Ne (α,γ) $^{20}_{10}$Ne $+ \frac{4}{2}α \rightarrow \frac{24}{12}$Mg

(e) $^{15}_{7}$N (p,α) $^{15}_{7}$N $+ \frac{1}{2}P \rightarrow \frac{4}{2}α + \frac{12}{6}$C

(f) $^{10}_{5}$B (n,α) $^{10}_{5}$B $+ \frac{1}{2}n \rightarrow \frac{4}{2}α + \frac{7}{3}$Li

12. Briefly describe the parts and the operation of a light water reactor.

A light water reactor uses fission of $^{235}$U to run the reactor. $^{235}$U is split most efficiently with slow-moving neutrons; however, the neutrons produced from fission are fast moving. A moderator composed of water is used to slow down the fast-moving neutrons.

$^{235}$U is present in only 0.7% of the naturally occurring U. To operate a reactor efficiently, the $^{235}$U must be enriched to a concentration of 3-4%.

Control rods are used to control reactor operations and to stop reaction. These are composed of substances (usually boron and/or cadmium) needed to absorb neutrons produced from the fission of the fuel.

13. Repeat Exercise 12 for the heavy water reactor.

This reactor works more or less the same as a light water reactor. Heavy water, D$_2$O, is used as the moderator. It is not as efficient as H$_2$O at slowing down the neutrons. However, since the neutrons move faster, they also move farther. This means the fuel does not need to be enriched. 0.7% $^{235}$U is concentrated enough in this reactor. The primary disadvantage is that it is expensive to produce D$_2$O.
14. Repeat Exercise 12 for the breeder reactor.

There are two disadvantages to using $^{235}$U as fuel for a reactor. First, known supplies of $^{235}$U may well run out within the next 50 years. Second, the fuel runs out, and new fuel rods added every few years. Breeder reactors operate using a different fuel. In this case, $^{238}$U, which does not undergo fission, is transmuted to form $^{239}$Pu. $^{239}$Pu does undergo fission and is the fuel for these reactors.

This process occurs at much higher temperatures, so water cannot be used as the moderator. Instead, liquid sodium is used as the moderator. Sodium is one of the most reactive substance known, and is thus corrosive to the walls of the container.

These reactors breed their own fuel. After an induction period of about 5-7 years, the reactor has produced enough fuel to maintain its own operations, and to fuel another reactor. Thus, these reactors are excellent power sources for satellites, deep-space exploration equipment, and other projects that a need long-term supply.

One distinct disadvantage is that $^{239}$Pu is one of the most toxic substances known to man, and it has a half-life of 24,000 years.

15. How does nuclear fission differ from nuclear fusion?
(a) Fission is the splitting of nuclei into lighter elements while fusion is the melting of nuclei.
(b) Fission is the splitting of a neutron while fusion is the combination of an electron and a proton to form a neutron.
(c) Fission is the splitting of nuclei into lighter elements while fusion is the combination of protons and neutrons in the nucleus.
(d) Fission is the splitting of nuclei into lighter elements while fusion is the consolidation of light nuclei to form heavier nuclei.

16. Briefly describe a fusion reactor. What advantages would it have over traditional fission reactors? What is the greatest obstacle that must be overcome in developing a fusion reactor?

These reactor produce energy by fusion of small nuclei, to form bigger nuclei. To overcome the initial repulsion of nuclei, the reactor must operate at extremely high temperatures of around 100 million °C. The practical problem is that there is no structural material able to withstand these temperatures. However, at these temperatures, all matter exists in a plasma state, which is a gaseous mixture of positive ions and electrons. It is considered possible to contain the plasma within a magnetic field. The technology is still in its infancy.

Some of the advantages of a fusion reactor are as follows: (a) a greater energy output is possible; (b) the wastes from these reactors is relatively clean; and (c) there is a limitless supply of fuel.
17. The half-life of a radionuclide depends on
(a) temperature.
(b) concentration.
(c) the amount present at the beginning of the reaction.
(d) none of the above.

18. The half-life of $^{19}$O is 29 s. What fraction of the isotope originally present would be left after 10.0 seconds?

\[ k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{29 \text{ s}} = 0.024 \text{ s}^{-1} \]

\[ \ln \left( \frac{[N]_0}{[N]} \right) = kt \]

\[ \ln \left( \frac{1}{[N]} \right) = (0.024 \text{ s}^{-1})(10.0 \text{ s}) \]

\[ \ln \left( \frac{1}{[N]} \right) = 0.24 \]

\[ \left( \frac{1}{[N]} \right) = e^{0.24} \]

\[ \left( \frac{1}{[N]} \right) = 1.27 \]

\[ [N] = \frac{1}{1.27} = 0.79 \]

19. The half-life of $^{11}$C is 20.3 min. (a) How long will it take for 95.0% of the sample to decay? (b) How long will it take for 99.5% of the sample to decay?

\[ k = \frac{\ln 2}{20.3 \text{ min}} = 0.0341 \text{ min}^{-1} \]

(a) \[ \ln \left( \frac{100}{5.0} \right) = (0.0341 \text{ min}^{-1})t \]

\[ 3.0 = (0.0341 \text{ min}^{-1})t \]

\[ t = \frac{3.0}{0.0341 \text{ min}^{-1}} = 88 \text{ min for 95\% to disappear} \]

(b) \[ \ln \left( \frac{100}{0.5} \right) = (0.0341 \text{ min}^{-1})t \]

\[ 5.3 = (0.0341 \text{ min}^{-1})t \]

\[ t = \frac{5.3}{0.0341 \text{ min}^{-1}} = 155 \text{ min for 99.5\% to disappear} \]
20. The activity of a sample of tritium decreased by 5.5% over a period of a year. What is the half-life of $^3$H?

$$\ln\left(\frac{100}{94.5}\right) = k(1.00 \text{ yr})$$

$$0.0566 = k(1.00 \text{ yr})$$

$$k = \frac{0.0566}{1.00 \text{ yr}} = 0.0566 \text{ yr}^{-1}$$

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{0.0566 \text{ yr}^{-1}} = 12.2 \text{ yr}$$

21. The $^{14}$C activity of an artifact from a burial site was 8.6 min·g C (that is 8.6 disintegrations per minute per gram C). The half-life of $^{14}$C is 5730 yrs. The current activity of $^{14}$C is 15.3 min·g C. How old is the artifact?

$$k = \frac{\ln 2}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

$$\ln\left(\frac{15.3}{8.6}\right) = (1.21 \times 10^{-4} \text{ yr}^{-1})t$$

$$0.576 = (1.21 \times 10^{-4} \text{ yr}^{-1})t$$

$$t = \frac{0.576}{1.21 \times 10^{-4} \text{ yr}^{-1}} = 4760 \text{ years}$$