You have studied the method known as "completing the square" to solve quadratic equations. Another use for this method is in transforming the equation for a circle or parabola into their standard forms.

The **standard form for equation of a circle** with center \((h, k)\) and radius \(r\) is

\[
(x - h)^2 + (y - k)^2 = r^2
\]

The **standard form for the equation of a parabola** with vertex \((h, k)\) is

\[
y = a(x - h)^2 + k
\]

If \(a > 0\), the parabola opens upward. If \(a < 0\), the parabola opens downward.

**COMPLETING THE SQUARE TO WRITE AN EQUATION FOR A CIRCLE IN STANDARD FORM:**

**Example 1:** Given the following equation for a circle, find its center and radius:

\[
x^2 + y^2 - 6x + 10y = 0
\]

**Solution:** We rearrange the terms so that those with \(x\) are together and those with \(y\) are together. We enclose them in parentheses, leaving room to "complete the square".

\[
(x^2 - 6x ) + (y^2 + 10y) = 0
\]

In each set of parentheses, add a constant that is the square of half the coefficient of the first degree term. That is, in the first parentheses add the square of \(-3\) (half of \(-6\)) and in the second parentheses add the square of 5 (half of +10). Compensate by adding the same on the right side of the equation.

\[
(x^2 - 6x + (-3)^2) + (y^2 + 10y + (5)^2) = 0 + (-3)^2 + (5)^2
\]

Factor the expressions on the left and simplify the right side to get

\[
(x - 3)^2 + (y + 5)^2 = 34
\]

Comparing this to the standard form, above, we can see that \(h = 3, k = -5\), and \(r^2 = 34\).

So this is an equation for a circle with center \((3, -5)\) and radius \(r = \sqrt{34} = 5.83\).
Example 2: Given the equation for a circle below, write its equation in standard form.

\[2x^2 + 2y^2 - 12y - 9 = 0\]

Solution: To use the method of completing the square, the coefficients of \(x^2\) and \(y^2\) need to be +1. We can divide through the equation by that coefficient. In this case, we divide through by 2:

\[\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{12y}{2} - \frac{9}{2} = \frac{0}{2}\]

Simplifying gives:

\[x^2 + y^2 - 6y - \frac{9}{2} = 0\].

Next, we need to rearrange the terms, grouping together terms with \(x\), grouping terms with \(y\), and the constant term will be moved to the right side:

\[\left(x^2\right) + \left(y^2 - 6y\right) = \frac{9}{2}\]

Finally, we will complete the square for \(x\) and \(y\). Since there is no first degree term in \(x\), we do not need to complete the square for \(x\). We replace \(\left(x^2\right)\) with \(\left(x - 0\right)^2\) which is equivalent and in the correct form. Completing the square for \(y\) requires that we add the square of half the coefficient of \(y\), the square of half of \(-3\), to both sides of the equation:

\[\left(x - 0\right)^2 + \left(y^2 - 6y + (-3)^2\right) = \frac{9}{2} + (-3)^2\]

Factoring on the left and simplifying on the right, gives

\[(x - 0)^2 + (y - 3)^2 = \frac{27}{2},\]

which is the standard form that was needed. Comparing this to the standard form of a circle, we see that \(h = 0\), \(k = 3\), and \(r^2 = \frac{27}{2}\).

So this is an equation for a circle with center \((0, 3)\) and radius \(r = \frac{\sqrt{27}}{2} \approx 3.67\).
COMPLETING THE SQUARE TO WRITE AN EQUATION FOR A PARABOLA IN STANDARD FORM:

Example 3: Transform the equation for the parabola \( y = x^2 - 6x + 5 \) into standard form. Determine the vertex of the parabola and which way it opens.

Solution: We only need to complete the square on the variable \( x \) in the equation of a parabola. The coefficient of \( x^2 \) should be +1. If it is not, we will divide through the equation by that coefficient. In this equation, the coefficient of \( x^2 \) is already +1, so we do not have to do this division. We can go on to the grouping process, grouping the \( x^2 \) and \( x \) terms together and moving the constant to the left, with the \( y \) variable.

\[
y - 5 = (x^2 - 6x)
\]

In the space left in the parentheses, we add the square of \(-3\) (\(-3\) is half of the coefficient of \( x \)). Of course, we also must add it to the other side of the equation.

\[
y - 5 + (-3)^2 = (x^2 - 6x + (-3)^2)
\]

Simplify the left side and factor the right side to get

\[
y + 4 = (x - 3)^2
\]

Finally, we isolate \( y \) by adding \(-4\) to each side which gives the standard form:

\[
y = (x - 3)^2 - 4
\]

Comparing this equation to the standard form for the equation of a parabola, the value of \( h \) is 3, the value of \( k \) is \(-4\), and \( a = +1 \) so the vertex is the point \((3, -4)\) and the parabola opens upward.

Example 4: For the parabola defined by the equation \( y = 3x^2 - 12x + 17 \), write the equation in standard form.

Solution: The coefficient of \( x^2 \) is not +1, so we begin by dividing through the equation by that coefficient, 3.

\[
\frac{y}{3} = \frac{3x^2}{3} - \frac{12x}{3} + \frac{17}{3}
\]

Simplifying gives

\[
\frac{y}{3} = x^2 - 4x + \frac{17}{3}
\]
Next, we group the $x^2$ and $x$ terms on the right, moving the constant term to the left side, with the $y$-term.

\[
\frac{y}{3} - \frac{17}{3} = (x^2 - 4x) 
\]

Add $(-2)^2$ to the group ($-2$ is half the coefficient of $x$) and add it to the left side of the equation.

\[
\frac{y}{3} - \frac{17}{3} + (-2)^2 = (x^2 - 4x + (-2)^2) 
\]

Simplify on the left and factor the right side to get

\[
\frac{y}{3} - \frac{5}{3} = (x - 2)^2 
\]

To put this into standard form we must isolate $y$. First add $\frac{5}{3}$ to both sides of the equation:

\[
\frac{y}{3} = (x - 2)^2 + \frac{5}{3} 
\]

then multiply both sides by 3:

\[
\frac{3}{1} \cdot \frac{y}{3} = 3 \cdot (x - 2)^2 + 3 \cdot \frac{5}{3} 
\]

which gives us

\[
y = 3 \cdot (x - 2)^2 + 5, 
\]

the standard form for the equation of the parabola. Comparing this to the standard form of a parabola, we see that $h = 2$ and $k = 5$, so the vertex of this parabola is $(2, 5)$. Also, $a = +3$ so the parabola opens upward.

**AN ALTERNATIVE METHOD FOR WORKING WITH PARABOLAS:**

In addition to being written in standard form as above, quadratic functions (whose graphs are parabolas that open up or down) may be written in the form $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$, where $a \neq 0$. We will now examine quadratic functions written in this form. Note that like quadratic equations written in standard form, if $a > 0$, the parabola opens upward. If $a < 0$, the parabola opens downward.
Example 5: For the function \( f(x) = -x^2 + 8x - 15, \)

a) Determine whether the parabola opens upward or downward.

b) Find the axis of symmetry.

c) Find the vertex, and find the maximum or minimum value.

d) Find the y-intercept.

e) Find the x-intercepts, if any.

f) Sketch the graph.

a) Since \( a, \) the coefficient of \( x^2, \) is \(-1\) (a *negative*), the parabola opens downward.

b) The axis of symmetry is the imaginary vertical line passing through the center of the parabola and may be found by using the formula \( x = -\frac{b}{2a} \) or \( x = \frac{-b}{2a}. \)

Note that if the graph is folded along the axis of symmetry, the two sides of the graph will lie directly on each other. In this problem, the axis of symmetry is \( x = \frac{8}{2(-1)} = -\frac{8}{-2} = -(4) = 4, \) or written simply \( x = 4. \) Since the axis of symmetry is a vertical line, your answer must include "\( x = \)" to indicate that it is the equation of a vertical line.

c) The vertex of the parabola always lies on the axis of symmetry and is the point \( \left( -\frac{b}{2a}, f\left( -\frac{b}{2a}\right) \right). \) The x-coordinate of the vertex may always be found in the equation of the axis of symmetry: since the axis of symmetry is \( x = 4, \) the vertex is \( (4, f(4)). \) Find \( f(4) \) to find the y-coordinate of the vertex:

\[
f(4) = -(4)^2 + 8(4) - 15 = -16 + 32 - 15 = 1
\]

Therefore, the vertex is \( (4, 1). \) Since the parabola opens downward, it has a *maximum* y-value at the vertex: \( f(4) = 1, \) which means that the highest y-value on the graph is 1.

d) Each quadratic function has exactly one y-intercept. To find the y-intercept, set \( x = 0 \) and solve for \( y. \) In other words, find \( f(0): \)

\[
f(0) = -(0)^2 + 8(0) - 15 = 0 + 0 - 15 = -15
\]

The y-intercept is \(-15\) or written as a point is \( (0, -15). \)

e) To find any x-intercepts (quadratic functions may have zero, one, or two x-intercepts), set \( y = 0 \) and solve for \( x. \) In other words, solve:

\[
0 = -x^2 + 8x - 15 \quad \text{(Solve by factoring or by using the quadratic formula.)}
\]

\[
0 = -(x^2 - 8x + 15) \quad \text{(Factor out \(-1\) so that the \( x^2 \) is positive.)}
\]

\[
0 = -(x - 5)(x - 3) \quad \text{(Factor the trinomial. Then set each factor equal to 0.)}
\]

\[
-1 \neq 0 \quad \text{(no variable in the factor \(-1\) so it doesn't give a solution)}
\]

\[
x - 5 = 0 \quad x = 5
\]

\[
x - 3 = 0 \quad x = 3
\]

Thus, the x-intercepts are 5 and 3. Written as points, they are \( (5, 0) \) and \( (3, 0). \)
f) To sketch the graph, plot the points you have found:
vertex (4,1), y-intercept (0, –15), and x-intercepts (5, 0) and (3, 0).
You may use a table of values and find more points as needed (the first four
points are those we have already found). For your table, you may want to select
x-values close to the x-coordinate of the vertex (choose x-values close to 4 in this
problem).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>–15</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
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</tr>
<tr>
<td>7</td>
<td>–8</td>
</tr>
<tr>
<td>8</td>
<td>–15</td>
</tr>
</tbody>
</table>

Example 6: For the function \( f(x) = 4x^2 - 6x - 3 \),
a) Determine whether the parabola opens upward or downward.
b) Find the axis of symmetry.
c) Find the vertex, and find the maximum or minimum value.
d) Find the y-intercept.
e) Find the x-intercepts, if any.
f) Sketch the graph.

a) Since \( a \), the coefficient of \( x^2 \), is positive 4, the parabola opens upward.

b) The axis of symmetry is
\[
x = -\frac{-6}{2(4)} = -\frac{-6}{8} = \frac{3}{4},
\]
or written simply \( x = \frac{3}{4} \). Again, since the axis of
symmetry is a vertical line, your answer must include "x =" to indicate that it is the
equation of a vertical line.

c) The vertex of the parabola always lies on the axis of symmetry and is the point
\[
\left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right).
\]
The x-coordinate of the vertex may always be found in the
equation of the axis of symmetry: since the axis of symmetry is $x = \frac{3}{4}$, the vertex is $(\frac{3}{4}, f(\frac{3}{4}))$. Find $f(\frac{3}{4})$ to find the y-coordinate of the vertex:

$$f(\frac{3}{4}) = 4(\frac{3}{4})^2 - 6(\frac{3}{4}) - 3 = 4(\frac{9}{16}) - \frac{9}{2} - \frac{3}{1} = \frac{9}{4} - \frac{18}{4} - \frac{12}{4} = \frac{-21}{4} \text{ or } -5 \frac{1}{4}.$$

Therefore, the vertex is $(\frac{3}{4}, -5 \frac{1}{4})$. Since the parabola opens upward, it has a **minimum** y-value at the vertex: $f(\frac{3}{4}) = -5 \frac{1}{4}$, which means that the lowest y-value on the graph is $-5 \frac{1}{4}$.

d) To find the y-intercept, set $x = 0$ and solve for $y$. In other words, find $f(0)$:

$$f(0) = 4(0)^2 - 6(0) - 3 = 0 - 0 - 3 = -3$$

The y-intercept is $-3$ or written as a point is $(0, -3)$.

e) To find any x-intercepts, set $y = 0$ and solve for $x$. In other words, solve:

$$0 = 4x^2 - 6x - 3 \text{ (Since this won't factor, use the quadratic formula to solve.)}$$

$$x = \frac{-(-6)\pm\sqrt{(-6)^2 - 4(4)(-3)}}{2(4)} = \frac{6 \pm \sqrt{36 + 48}}{8} = \frac{6 \pm \sqrt{84}}{8}.$$ 

Now simplify the square root:

$$\frac{6 \pm \sqrt{84}}{8} = \frac{6 \pm \sqrt{4\times21}}{8} = \frac{6 \pm 2\sqrt{21}}{8}.$$ 

Finally, simplify (reduce) the fraction by factoring numerator (and denominator) and canceling the common 2:

$$x = \frac{6 \pm 2\sqrt{21}}{8} = \frac{2(3 \pm \sqrt{21})}{2(4)} = \frac{3 \pm \sqrt{21}}{4}.$$ 

This gives two answers: $\frac{3 + \sqrt{21}}{4} \approx 1.9$ and $\frac{3 - \sqrt{21}}{4} \approx -0.4$.

Note that we could have also written each term in the numerator separately over the denominator and then simplified:

$$x = \frac{6 \pm 2\sqrt{21}}{8} = \frac{6}{8} \pm \frac{2\sqrt{21}}{8} = \frac{3}{4} \pm \frac{\sqrt{21}}{4}$$ 

which is another way to write the exact answer. This form of the exact answer gives the same decimal approximations as the other. The exact x-intercepts are
\[
\frac{3 + \sqrt{21}}{4} \quad \text{and} \quad \frac{3 - \sqrt{21}}{4}
\]. Written as points with approximate x-coordinates, they are (1.9, 0) and (−0.4, 0).

f) To sketch the graph, plot the points you have found:

vertex \( \left( \frac{3}{4}, -\frac{5}{4} \right) \), y-intercept (0, −3), and approximate x-intercepts (1.9, 0) and (−0.4, 0). Use a table of values to find more points as needed (the first four points in the table are those we have already found). For your table, you may want to select x-values close to the x-coordinate of the vertex (choose x-values close to \( \frac{3}{4} \) in this problem).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4} )</td>
<td>−( \frac{5}{4} )</td>
</tr>
<tr>
<td>0</td>
<td>−3</td>
</tr>
<tr>
<td>1.9</td>
<td>0</td>
</tr>
<tr>
<td>−0.4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>−5</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>−1</td>
<td>7</td>
</tr>
</tbody>
</table>

Example 7: For the function \( f(x) = x^2 + 4x + 7 \),

a) Determine whether the parabola opens upward or downward.

b) Find the axis of symmetry.

c) Find the vertex, and find the maximum or minimum value.

d) Find the y-intercept.

e) Find the x-intercepts, if any.

f) Sketch the graph.

a) Since \( a \), the coefficient of \( x^2 \), is +1, the parabola opens upward.

b) The axis of symmetry is

\[
x = -\frac{4}{2(1)} = -\frac{4}{2} = -2,
\]

or written simply \( x = -2 \). Again, since the axis of symmetry is a vertical line, your answer must include "x =" to indicate that it is the equation of a vertical line.

c) The vertex of the parabola always lies on the axis of symmetry and is the point \( \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \). The x-coordinate of the vertex may always be found in the
equation of the axis of symmetry: since the axis of symmetry is $x = -2$, the vertex is $(-2, f(-2))$. Find $f(-2)$ to find the y-coordinate of the vertex:

$$f(-2) = (-2)^2 + 4(-2) + 7 = 4 - 8 + 7 = 3$$

Therefore, the vertex is $(-2, 3)$. Since the parabola opens upward, it has a minimum y-value at the vertex: $f(-2) = 3$, which means that the lowest y-value on the graph is 3.

d) To find the y-intercept, set $x = 0$ and solve for $y$. In other words, find $f(0)$:

$$f(0) = (0)^2 + 4(0) + 7 = 0 + 0 + 7 = 7$$

The y-intercept is 7 or written as a point is $(0, 7)$.

e) To find any x-intercepts, set $y = 0$ and solve for $x$. In other words, solve:

$$0 = x^2 + 4x + 7$$

(Since this won't factor, use the quadratic formula to solve.)

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(7)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 28}}{2} = \frac{-4 \pm \sqrt{-12}}{2}$$

Since the discriminant, $-12$, is negative (i.e., $\sqrt{-12}$ is not a real number), this equation has no real number solutions and therefore we have no x-intercepts. Note that the vertex is above the x-axis and the parabola opens up. When this is the case (or when the vertex is below the x-axis and the parabola opens down), the graph will have no x-intercepts because it never intersects the x-axis. When the discriminant is zero, the graph will have only one x-intercept. When the discriminant is positive, the graph will have two x-intercepts.

f) To sketch the graph, plot the points you have found: vertex $(-2, 3)$, and y-intercept $(0, 7)$.

Use a table of values to find more points as needed (the first two points in the table are those we have already found). For your table, you may want to select x-values close to the x-coordinate of the vertex (choose x-values close to $-2$ in this problem).

<table>
<thead>
<tr>
<th>$x$</th>
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</tr>
</thead>
<tbody>
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</tr>
<tr>
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</tr>
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<td>-5</td>
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<tr>
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<td>12</td>
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</table>
APPLICATIONS OF QUADRATIC EQUATIONS:

There are applications of quadratic equations other than those found in Lesson 10.2. Below are a couple of examples of other types of quadratic applications.

Example 8: For an experiment, a ball is projected with an initial velocity of 80 feet/sec. Neglecting air resistance, its height \( h \), in feet, after \( t \) seconds is given by the formula

\[
h = 80t - 16t^2
\]

How long will it take for the ball to hit the ground?

Solution: When the ball hits the ground, its height \( h \) is 0 feet. To solve this problem, substitute 0 in place of \( h \) and solve for \( t \) by factoring or by using the quadratic formula:

Substitute 0 in place of \( h \): \[0 = 80t - 16t^2\]
Factor the right side of the equation: \[0 = 16t(5 - t)\]
Set each factor equal to zero: \[16t = 0 \text{ or } 5 - t = 0\]
Solve each equation to get: \[t = 0 \text{ or } t = 5\]

Note that \( t = 0 \) seconds when the ball is first projected from the ground, so we exclude this answer (note that we would also exclude any negative times). The other answer \( t = 5 \) indicates that the ball will hit the ground 5 seconds after it is projected.

Example 9: Nigel throws an object upward from the top of a building. The distance \( d \), in ft, of the object from the ground at any time \( t \), in seconds, can be found by the formula

\[
d = -16t^2 + 96t + 64
\]

a) Find the time the object reaches its maximum height.

b) Find the maximum height.

Solution: a) Note that the equation given in the problem represents a parabola with time \( t \) on the x-axis and distance \( d \) on the y-axis. The parabola opens downward because the coefficient of \( t^2 \) is negative. The maximum point on a downward parabola is the vertex. Find the answers to both parts of this problem by finding the vertex of the parabola. Use \( t = \frac{-b}{2a} \) to find the time it reaches its maximum height:

\[
t = \frac{-96}{2(-16)} = \frac{-96}{-32} = 3 \text{ seconds}
\]

b) Substitute your answer into the equation in place of \( t \) to find the maximum height:

\[
d = -16(3)^2 + 96(3) + 64 = -144 + 288 + 64 = 208 \text{ feet}
\]
Complete the square where necessary to write the equations for the following circles in standard form. Identify the center and the radius of each circle. Round radius to two decimal places where appropriate.

1. \( x^2 + y^2 + 8x - 6y - 15 = 0 \)
2. \( x^2 + y^2 - 8x + 2y + 13 = 0 \)
3. \( x^2 + y^2 + 10y - 75 = 0 \)
4. \( x^2 + y^2 + 7x - 3y - 10 = 0 \)
5. \( 16x^2 + 16y^2 = 1 \)

Complete the square where necessary to write the equations for the following parabolas in standard form. Identify the vertex of each parabola.

6. \( y = x^2 + 2x + 3 \)
7. \( y = -x^2 + 4x - 5 \)
8. \( y = x^2 - 2x + 1 \)
9. \( y = 2x^2 + 4x - 1 \)
10. \( y = -x^2 - 6x - 5 \)

For each function below, a) determine whether the parabola opens up or down, b) find the axis of symmetry, c) find the vertex and find the maximum or minimum value, d) find the y-intercept, e) find the x-intercepts, if any, and f) sketch the graph.

11. \( f(x) = x^2 - 6x + 8 \)
12. \( f(x) = -x^2 + 4x - 9 \)
13. \( f(x) = x^2 - 3x + 7 \)
14. \( f(x) = x^2 + 6x + 2 \)
15. \( f(x) = -2x^2 - 6x + 4 \)
16. \( f(x) = -3x^2 + 6x - 3 \)
17. A rocket is projected from a height 9.8 meters above the ground. The height \( h \), in meters, of the ball above the ground at any time \( t \), in seconds, is determined by the formula

\[
h = -4.9t^2 + 24.5t + 9.8
\]

a) Find the time the rocket reaches its maximum height.
b) Find the maximum height.

18. A weightlifter throws a ball upward from the top of a building. The height \( h \), in ft, of the object from the ground at any time \( t \), in seconds, can be found using the formula

\[
h = -16t^2 + 48t + 64
\]

Find the time the ball will strike the ground.

19. An object is projected upward with an initial velocity of 192 feet per second. The distance \( d \), in feet, above the ground at any time \( t \), in seconds is determined by the formula

\[
d = -16t^2 + 192t
\]

Find the time the object will hit the ground.

20. A baseball player throws a ball into the air with initial velocity of 32 feet per second. The distance \( d \), in feet, above the ground at any time \( t \), in seconds, is given by the formula

\[
d = 32t - 16t^2
\]

a) Find the time the ball reaches its maximum distance above the ground.
b) Find the maximum distance above the ground.
ANSWERS:

1. Equation of Circle in Standard Form: \((x + 4)^2 + (y - 3)^2 = 40\)
   Center: \((-4, 3)\)
   Radius: \(\sqrt{40} \approx 6.32\)

2. Equation of Circle in Standard Form: \((x - 4)^2 + (y + 1)^2 = 4\)
   Center: \((4, -1)\)
   Radius: \(\sqrt{4} = 2\)

3. Equation of Circle in Standard Form: \((x - 0)^2 + (y + 5)^2 = 100, or \ x^2 + (y + 5)^2 = 100\)
   Center: \((0, -5)\)
   Radius: \(\sqrt{100} = 10\)

4. Equation of Circle in Standard Form: \(\left(x + \frac{7}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{49}{2}\)
   Center: \((-\frac{7}{2}, \frac{3}{2})\)
   Radius: \(\sqrt{\frac{49}{2}} \approx 4.95\)

5. Equation of Circle in Standard Form: \((x - 0)^2 + (y - 0)^2 = \frac{1}{16}, or \ x^2 + y^2 = \frac{1}{16}\)
   Center: \((0, 0)\)
   Radius: \(\sqrt{\frac{1}{16}} = \frac{1}{4}\)

6. Equation of Parabola in Standard Form: \(y = (x + 1)^2 + 2\)
   Vertex: \((-1, 2)\)

7. Equation of Parabola in Standard Form: \(y = -(x - 2)^2 - 1\)
   Vertex: \((2, -1)\)

8. Equation of Parabola in Standard Form: \(y = (x - 1)^2 + 0\)
   Vertex: \((1, 0)\)

9. Equation of Parabola in Standard Form: \(y = 2(x + 1)^2 - 3\)
   Vertex: \((-1, -3)\)

10. Equation of Parabola in Standard Form: \(y = -(x + 3)^2 + 4\)
    Vertex: \((-3, 4)\)
ANSWERS:

11. a) opens up
   b) axis of symmetry: $x = 3$
   c) vertex: $(3, -1)$; Min $f(3) = -1$
   d) $y$-intercept: $(0, 8)$
   e) $x$-intercept(s): $(2, 0)$ and $(4, 0)$
   f) 
   
   
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

12. a) opens down
   b) axis of symmetry: $x = 2$
   c) vertex: $(2, -5)$; Max $f(2) = -5$
   d) $y$-intercept: $(0, -9)$
   e) $x$-intercept(s): None
   f) 
   
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-14</td>
</tr>
<tr>
<td>4</td>
<td>-9</td>
</tr>
</tbody>
</table>

13. a) opens up
   b) axis of symmetry: $x = \frac{3}{2}$
   c) vertex: $\left(\frac{1}{2}, \frac{3}{4}\right)$; Min $f\left(\frac{1}{2}\right) = \frac{3}{4}$
   d) $y$-intercept: $(0, 7)$
   e) $x$-intercept(s): None
   f) 
   
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>4.75</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>
14. a) opens up
   b) axis of symmetry: \( x = -3 \)
   c) vertex: \((-3, -7\)); Min \( f(-3) = -7 \)
   d) y-intercept: \((0, 2)\)
   e) x-intercept(s): \((-3 + \sqrt{7}, 0\) and \((-3 - \sqrt{7}, 0\) )
   f)

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-3 & -7 \\
0 & 2 \\
-0.4 & 0 \\
-5.6 & 0 \\
-4 & -6 \\
-5 & -3 \\
-6 & 2 \\
-2 & -6 \\
-1 & -3 \\
1 & 9 \\
\hline
\end{array}
\]

15. a) opens down
   b) axis of symmetry: \( x = -\frac{3}{2} \)
   c) vertex: \((-1\frac{1}{2}, 8\frac{1}{2}\)); Max \( f(-1\frac{1}{2}) = 8\frac{1}{2} \)
   d) y-intercept: \((0, 4)\)
   e) x-intercept(s): \((-3 + \frac{\sqrt{17}}{2}, 0\) and \((-3 - \frac{\sqrt{17}}{2}, 0\) )
   f)

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-1.5 & 8.5 \\
0 & 4 \\
0.6 & 0 \\
-3.6 & 0 \\
1 & -4 \\
-1 & 8 \\
-2 & 8 \\
-3 & 4 \\
\hline
\end{array}
\]
ANSWERS:

16. a) opens down  
   b) axis of symmetry: \( x = 1 \)  
   c) vertex: \((1, 0)\); Max \( f(1) = 0 \)  
   d) y-intercept: \((0, -3)\)  
   e) x-intercept(s): \((1, 0)\)  
   f)  
   \[
   \begin{array}{|c|c|}
   \hline
   x & y \\
   \hline
   1 & 0 \\
   0 & -3 \\
   -1 & -12 \\
   2 & -3 \\
   3 & -12 \\
   \hline
   \end{array}
   \]

17. a) The rocket reaches its maximum height in 2.5 seconds.  
   b) Its maximum height is 40.425 meters.

18. The ball will strike the ground in 4 seconds.

19. The object will hit the ground in 12 seconds.

20. a) The ball reaches its maximum height in 1 second.  
   b) Its maximum height is 16 feet.