Conic Sections

The graph of $Ax^2 + By^2 + Cx + Dy + E = 0$, if it exists, is

- a **parabola** if either $A$ or $B$, but not both, equal 0;
- a **circle** if both $A$ and $B$ are nonzero and $A=B$;
- an **ellipse** if both $A$ and $B$ are nonzero and $A$ and $B$ have the same sign, but $A \neq B$;
- a **hyperbola** if both $A$ and $B$ are nonzero and $A$ and $B$ have opposite signs.

\[
y - k = a(x - h)^2
\]

**parabola** that opens up if $a > 0$ and opens down if $a < 0$
vertex: $(h,k)$
axis of symmetry: $x = h$

\[
x - h = a(y - k)^2
\]

**parabola** that opens right if $a > 0$ and opens left if $a < 0$
vertex: $(h,k)$
axis of symmetry: $y = k$

\[
(x - h)^2 + (y - k)^2 = r^2
\]

**circle**
center: $(h,k)$ radius: $r$

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]

**ellipse**
center: $(h,k)$
horizontal axis: extends $a$ units to the left and right of center
vertical axis: extends $b$ units above and below the center

\[
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
\]

**hyperbola** with branches opening to the left and right
center: $(h,k)$
the branches pass through points $a$ units to the left and right of the center
the branches approach the asymptotes: $y - k = \pm \frac{b}{a}(x - h)$ as $x \to \pm \infty$

\[
\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1
\]

**hyperbola** with branches opening up and down
center: $(h,k)$
the branches pass through points $b$ units above and below the center
the branches approach the asymptotes: $y - k = \pm \frac{b}{a}(x - h)$ as $y \to \pm \infty$