Odd and Even $x$-Intercept Properties of Polynomial Graphs

If a factor of a polynomial occurs to an odd power, then the graph of the polynomial goes across the $x$-axis at the corresponding $x$-intercept. An $x$-intercept of this type is sometimes called an odd $x$-intercept. If a factor of a polynomial occurs to an even power, then the graph of the polynomial bounces against the $x$-axis at the corresponding $x$-intercept, but not does not go across the $x$-axis there. An $x$-intercept of this type is sometimes called an even $x$-intercept.

**Example:** Use a graphing calculator to graph $y = 0.01x^2(x + 2)^3(x - 2)(x - 4)^4$.

Because the factors $(x + 2)$ and $(x - 2)$ appear to odd powers, the graph crosses the $x$-axis at $x = -2$ and $x = 2$.

Because the factors $x$ and $(x - 4)$ appear to even powers, the graph bounces against the $x$-axis at $x = 0$ and $x = 4$.

Note that if the factors of the polynomial were multiplied out, the leading term would be $0.01x^{10}$. This accounts for the fact that both tails of the graph go up; in other words, as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to \infty$.

The odd and even $x$-intercept properties along with what we know about the end behavior (the tails) of a polynomial enable us to draw a rough sketch of the polynomial’s graph. The following exercises will provide practice at doing that.

1. Let $f(x) = -(x - 3)(x + 2)^2$.
   (a) Determine the $x$-intercepts.
   (b) Determine the $y$-intercept.
   (c) Describe the behavior of $y$ as $x \to -\infty$.
   (d) Describe the behavior of $y$ as $x \to \infty$.
   (e) Without using your calculator, draw a rough sketch of $y = f(x)$.
   (f) Solve $-x(x - 3)(x + 2)^2 \leq 0$. Give the solution in interval notation.

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2. Let \( f(x) = -x^5 + 6x^3 - 5x \).
(a) Determine the \( x \)-intercepts.
(b) Determine the \( y \)-intercept.
(c) Describe the behavior of \( y \) as \( x \to -\infty \).
(d) Describe the behavior of \( y \) as \( x \to \infty \).
(e) Without using your calculator, draw a rough sketch of
\( y = f(x) \).
(f) Solve \( -x^5 + 6x^3 - 5x > 0 \). Give the solution in interval notation.

3. Let \( f(x) = -(x - 2)^2(x + 4) \).
(a) Determine the \( x \)-intercepts.
(b) Determine the \( y \)-intercept.
(c) Describe the behavior of \( y \) as \( x \to -\infty \).
(d) Describe the behavior of \( y \) as \( x \to \infty \).
(e) Without using your calculator, draw a rough sketch of
\( y = f(x) \).
(f) Solve \( -(x - 2)^2(x + 4) > 0 \). Give the solution in interval notation.

4. Determine (a) \( x \)-intercepts and (b) end behavior as \( x \to -\infty \) for \( f(x) = 2x^4 - 12x^3 - 18x^2 \).
(c) Solve \( 2x^4 - 12x^3 - 18x^2 > 0 \) and give your answer using interval notation.

5. Determine (a) \( x \)-intercepts and (b) end behavior as \( x \to \infty \) for \( f(x) = 2x^3 - 3x^2 - 10x + 15 \).
(c) Solve \( 2x^3 - 3x^2 - 10x + 15 \leq 0 \) and give your answer using interval notation.

6. Determine (a) \( x \)-intercepts and (b) end behavior as \( x \to \infty \) for \( f(x) = -x^5 + 4x^4 - x^3 \).
(c) Solve \( -x^5 + 4x^4 - x^3 \geq 0 \) and give your answer using interval notation.

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