Activity 4.8 Compound Interest

Overview:
The activity explores the value of \( e \) and the relationship between \( ab^t \) and \( ae^{kt} \) by having students calculate the growth factor for an amount of money that is compounded more and more often.

Estimated Time Required: The activity should take approximately 15 minutes.

Technology: Scientific Calculator

Prerequisite Concepts:
- Compound Interest
- Growth factor
- Growth rate

Discussion:
Review converting between \( Q(t) = ab^t \) and \( Q(t) = ae^{kt} \), where \( b = e^k \), and \( k = \ln b \) and the general formula for any quantity that is growing or decaying at a continuous rate \( k \), \( Q(t) = ae^{kt} \), and note that \( e^k \) is the annual effective growth factor.

Point out that the compound interest formula \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \) is an example of an exponential formula with initial value \( P \) and growth factor \( \left( 1 + \frac{r}{n} \right)^n \). You may want to have students work several compound interest problems like #1, #3, and #5 in the exercise set.

Have student complete the activity and then discuss the fact that the effective rate is 22.14% for the nominal rate of 20% if there were an unlimited number of compounding periods. That is, the effective rate is \( e^{0.20} - 1 \).

Summarize the findings that \( e \) raised to the growth rate gives the annual effective growth factor, the growth factor minus 1 is the growth rate, and that \( A = Pe^{rt} \) is used for calculating compound interest when the quantity is growing or decaying at a continuous rate \( k \). Note that the formula works when \( r \) is negative, which indicates decay.
Activity 4.8 Continuous Growth and the Number e

Because any exponential function can be written as \( Q(t) = ab^t \) or as \( Q(t) = ae^{kt} \), where \( b = e^k \) and \( k = \ln b \), the two formulas represent the same function. We call \( b \) the growth factor, and we call \( k \) the continuous growth rate.

Consider the classic interest problem of an account earning a nominal interest rate of 20% per year, being compounded many times per year. Find the growth factor for the different number of compounding periods in a year and enter them in the following table:

<table>
<thead>
<tr>
<th>Number of Compounding Periods</th>
<th>Growth Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td></td>
</tr>
</tbody>
</table>

What happens as we continue increasing the number of compounding periods?

Does there appear to be a maximum possible effective rate we can earn? If so, what is it?

What is the relationship between \( e \) and the growth factors we calculated above?