Statistical Process Control

Pareto Charts

Pareto charts are bar graphs with the bars ordered by height. They are often used to isolate the “vital few” categories on which we should focus our attention. Exercise 27.4 of BPS describes the following example. A large medical center, financially pressed by restrictions on reimbursement by insurers and the government, looked at losses broken down by diagnosis. Government standards place cases into diagnostic related groups (DRGs). For example, major joint replacements (mostly hip and knee) are DRG 209. The data in Exercise 27.4 and EX27-04.MTW list the 9 DRGs with the most losses along with the percent of losses. Since the percents are given, a Pareto chart can be constructed by selecting Graph ➤ Bar Chart from the menu. Indicate that the bars represent “Values from a table” and select Simple.
In the following dialog box, indicate that the graph variable is 'Percent of Losses' and the categorical variable is DRG.

Click on the Bar Chart Options button and choose order based on Decreasing Y in the Bar Chart Options subdialog box. Click OK on the dialog boxes to make the Pareto chart. The Pareto chart helps the hospital decide which DRGs the hospital should study first when attempting to reduce its losses.
Control Charts for Sample Means

Example 27.3 of BPS discusses a manufacturer of computer monitors. The manufacturer measures the tension of fine wires behind the viewing screen. Tension is measured by an electrical device with output readings in millivolts (mV). The proper tension is 275 mV. Some variation is always present in the production process. When the process is operating properly, the standard deviation of the tension readings is $\sigma = 43$ mV. Four measurements are made every hour. Table 27.1 of BPS and TA27-01.MTW contain the measurements for 20 hours. The first row of observations is from the first hour, the next row is from the second hour, and so on. There are a total of 80 observations.
Minitab can be used to produce control charts for sample means by selecting

Stat ➤ Control Charts ➤ Variable Chart for Subgroups ➤ Xbar

from the menu. An $\bar{x}$-control chart can be made using either the raw sample data in columns C2–C5 as described in Chapter 11. To make a control chart using the $\bar{x}$ data, specify that the data are arranged as a Single column and select the column with the $\bar{x}$ data.

The $\bar{x}$ values will be plotted on the chart. In addition, a center line, an upper control limit (UCL) at $3\sigma$ above the center line, and a lower control limit (LCL) at $3\sigma$ below the center line are drawn on the chart. By default, the process mean $\mu$ and standard deviation $\sigma$ are estimated from the data. Alternatively, the parameters $\mu$ and $\sigma$ may be specified from historical data by clicking on the Xbar Options button and selecting the Parameters tab. In the following dialog box we specify that the historical mean is equal to 275 and the historical standard deviation is equal to $\sigma/\sqrt{n} = 21.5$. 

![Xbar Chart dialog box](image1)

![Xbar Chart Options dialog box](image2)
Alternatively, the $\bar{x}$-control chart can be made by specifying that the data are arranged as “Subgroups of rows of” and select the columns containing the raw data. In this case, the historical mean is still equal to 275, but the historical standard deviation is equal to $\sigma = 43$. As before, click on the Xbar Options button and the Parameters tab. Both methods will produce an $\bar{x}$-control chart (with a slightly different title), as follows.

In this $\bar{x}$ chart, no points lie outside the control limits. In practice, we must monitor both the process center, using an $\bar{x}$ chart, and the process spread, using a control chart for the sample standard deviation $s$.

In practice, we must control the center of a process and its variability. This is commonly done with an $s$ chart, a chart of standard deviations against time. The $s$ chart can be produced by selecting

Stat ➤ Control Charts ➤ Variables Charts for Subgroups ➤ S

from the menu. The dialog box is filled exactly the same way as the dialog box for the $\bar{x}$ chart on the previous page. Often, $s$ minus three standard deviations gives a negative number. In this case, the lower control limit is plotted at 0.

The $s$ chart for the mesh tension data is also in control. Usually, the $\bar{x}$ chart and the $s$ chart will be looked at together. We can produce both charts at once by selecting

Stat ➤ Control Charts ➤ Variables Charts for Subgroups ➤ Xbar-S

from the menu. As usual, we have a choice of specifying or not specifying the historical values of $\mu$ and $\sigma$. These affect the centerline and control limits of the $\bar{x}$ chart only.
Out-of-Control Signals

Minitab performs tests to identify out-of-control signals. Each test detects a specific pattern in the data plotted on the chart. The occurrence of a pattern suggests a special cause for the variation, one that should be investigated. The tests can be selected by clicking the Xbar Options button on the dialog box and choosing from the Tests tab.

When a point fails a test, it is marked with the test number on the plot. If a point fails more than one test, the number of the first test in your list is the number displayed on the plot.
Control Charts for Sample Proportions

Example 27.13 of PBS discusses using $p$ charts for manufacturing and school absenteeism. Table 27.8 of PBS and TA27-08.MTW contain data on production workers and record the number and proportion absent from work each day during a period of four weeks. Minitab can be used to produce control charts for proportions by selecting

**Stat ➤ Control Charts ➤ Attributes chart ➤ P**

from the menu. Minitab draws a $p$ chart to show the proportion absent (the number absent divided by the subgroup size). $P$ charts track the proportion absent and detects the presence of special causes. Each entry in the worksheet column is the number absent for one subgroup. In the dialog box under Variables, enter the column that contain the number absent (or defective) for each sample. Choose “Size” for subgroups, then enter the subgroup size as shown.

Choose ‘ID column’ for unequal-size subgroups, then enter a column of subscripts. If the subgroups are not equal, each control limit is not a single straight line but varies with the subgroup size.

Click on the P Chart Options button and the Parameters tab if you wish to enter a historical value for $p$. 
The $p$ chart shows a clear downward trend in the daily proportion of workers who are absent. It appears that actions were taken to reduce the absenteeism rate. The last two weeks’ data suggest that the rate has stabilized.

EXERCISES

27.10 A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. The hardness of a sample from each lot of tablets is measured in order to control the compression process. The process has been operating in control with mean at the target value $\mu = 11.5$ and estimated standard deviation $\sigma = 0.2$. Table 27.2 gives three sets of data, each representing $\bar{x}$ for 20 succes-
sive samples of \( n = 4 \) tablets. One set remains in control at the target value. In a second set, the process mean \( \mu \) shifts suddenly to a new value. In a third, the process mean drifts gradually.

(a) What are the center line and control limits for an \( \bar{x} \) chart for this process?

(b) Select Stat ➤ Control Charts ➤ Variables for Subgroups ➤ Xbar to draw separate \( \bar{x} \) charts for each of the three data sets. Mark any points that are beyond the control limits.

(c) Based on your work in (b) and the appearance of the control charts, which set of data comes from a process that is in control? In which case does the process mean shift suddenly and at about which sample do you think the mean changed? Finally, in which case does the mean drift gradually?

27.13 Exercise 27.10 concerns process control data on the hardness of tablets for a pharmaceutical product. Table 27.4 gives data for 20 new samples of size 4, with the \( \bar{x} \) and \( s \) for each sample. The process has been in control with mean at the target value \( \mu = 11.5 \) and standard deviation \( \sigma = 0.2 \).

(a) Select Stat ➤ Control Charts ➤ Variables for Subgroups ➤ Xbar-S to make both \( \bar{x} \) and \( s \) charts for these data based on the information given about the process.

(b) At some point, the within-sample process variation increased from \( \sigma = 0.2 \) to \( \sigma = 0.4 \). About where in the 20 samples did this happen? What is the effect on the \( s \) chart? On the \( \bar{x} \) chart?

(c) At that same point, the process mean changed from \( \mu = 11.5 \) to \( \mu = 11.7 \). What is the effect of this change on the \( s \) chart? On the \( \bar{x} \) chart?

27.15 Figure 27.10 reproduces a data sheet from the floor of a factory that makes electrical meters. The sheet shows measurements of the distance between two mounting holes for 18 samples of size 5. The heading informs us that the measurements are in multiples of 0.0001 inch above 0.6000 inch. That is, the first measurement, 44, stands for 0.6044 inch. All the measurements end in 4. Although we don’t know why this is true, it is clear that in effect the measurements were made to the nearest 0.001 inch, not to the nearest 0.0001 inch. EX27-15.MTW contains the data along with \( \bar{x} \) and \( s \) for all 18 samples. Based on long experience with this process, you are keeping control charts based on \( \mu = 43 \) and \( \sigma = 12.74 \). Select Stat ➤ Control Charts ➤ Variables for Subgroups ➤ Xbar-S to make \( s \) and \( \bar{x} \) charts for the data in Figure 27.10 and describe the state of the process.

27.21 Table 27.6 in BPS and TA27-06.MTW give data on the losses (in dollars) incurred by a hospital in treating DRG 209 (major joint replacement) patients. The hospital has taken from its records a random sample of 8 such patients each month for 15 months. Select Stat ➤ Control Charts ➤ Combination Charts ➤ Xbar-S from the menu to make an \( \bar{x} \) and \( s \) chart.
(a) Does the $s$ control chart show any points out of control? Is it save to base the $\bar{x}$ chart on all 15 samples?
(b) Is the $\bar{x}$ chart in control?

27.25 Table 27.6 of BPS and EX2406.MTW give data on hospital losses for samples of DRG 209 patients. Select **Stat ▶ Basic Statistics ▶ Display Descriptive Statistics** from the menu to obtain numerical summaries for the data. The distribution of losses has been stable over time. What are the natural tolerances within which you expect losses on nearly all such patients to fall?

27.26 Do the losses on the 120 individual patients in Table 27.6 in BPS and TA27-06.MTW appear to come from a single Normal distribution? Select **Graph ▶ Histogram** from the menu and select “With Fit” and discuss what the graph shows. Are the natural tolerances you found in the previous exercise trustworthy?

27.27 If the mesh tension of individual monitors follows a Normal distribution, we can describe capability by giving the percent of monitors that meet specifications. The old specifications for mesh tension are 100 to 400 mV. The new specifications are 150 to 350 mV. Because the process is in control, we can estimate that tension has mean 275 mV and standard deviation 38.4 mV. Select **Calc ▶ Probability Distributions ▶ Normal** to answer the following questions.

   (a) What percent of monitors meet the old specifications?
   (b) What percent meet the new specifications?
   (c) We can improve capability by adjusting the process to have center 250 mV. This is an easy adjustment that does not change the process variation. What percent of monitors now meet the new specifications?

27.31 After inspecting Figure 27.16 in BPS, you decide to monitor the next four weeks’ absenteeism rates using a center line and control limits calculated from the second two weeks’ data recorded in Table 27.8 and TA27-08.MTW. Find $\bar{p}$ for these 10 days. Select **Stat ▶ Control Charts ▶ Attributes chart ▶ P** from the menu. Click on the P Chart Options button and the Parameters tab to use this value to compute the CL, LCL, and UCL. (Until you have more data, these are trial control limits. As long as you are taking steps to improve absenteeism, you have not reached the process-monitoring stage.)

27.35 Here and in EX27-35.MTW are data from an urban school district on the number of eighth-grade students with three or more unexcused absences from school during each month of a school year. Because the total number of eighth graders changes a bit from month to month, these totals are also given for each month.
Select \textbf{Calc} \textgreater \textbf{Calculator} from the menu to find \( \overline{p} \). Because the number of students varies from month to month, also find \( \overline{n} \), the average per month.

(b) Select \textbf{Stat} \textgreater \textbf{Control Charts} \textgreater \textbf{Attributes chart} \textgreater \textbf{P} to make a \( p \) chart using control limits based on \( \overline{n} \) (rounded) students each month. Comment on control.

(c) The exact control limits are different each month because the number of students \( n \) is different each month. This situation is common in using \( p \) charts. Instead of using \( \overline{n} \) for the Subgroup “Size”, you can select a Subgroup “Indicator Column” when you make your chart. Does using exact limits affect your conclusions?

### 27.39
You manage the customer service operation for a maker of electronic equipment sold to business customers. Traditionally, the most common complaint is that equipment does not operate properly when installed, but attention to manufacturing and installation quality will reduce these complaints. You hire an outside firm to conduct a sample survey of your customers. Here and in EX27-39.MTW are the percents of customers with each of several kinds of complaints:

<table>
<thead>
<tr>
<th>Category</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy of invoices</td>
<td>25</td>
</tr>
<tr>
<td>Clarity of operating manual</td>
<td>8</td>
</tr>
<tr>
<td>Complete invoice</td>
<td>24</td>
</tr>
<tr>
<td>Complete shipment</td>
<td>16</td>
</tr>
<tr>
<td>Correct equipment shipped</td>
<td>15</td>
</tr>
<tr>
<td>Ease of obtaining invoice</td>
<td>33</td>
</tr>
<tr>
<td>adjustments/credits</td>
<td></td>
</tr>
<tr>
<td>Equipment operates when</td>
<td>6</td>
</tr>
<tr>
<td>installed</td>
<td></td>
</tr>
<tr>
<td>Meeting promised delivery date</td>
<td>11</td>
</tr>
<tr>
<td>Sales rep returns calls</td>
<td>4</td>
</tr>
<tr>
<td>Technical competence of sales</td>
<td>12</td>
</tr>
<tr>
<td>rep</td>
<td></td>
</tr>
</tbody>
</table>

(a) Why do the percents not add to 100%?

(b) Select \textbf{Graph} \textgreater \textbf{Bar Chart} to make a Pareto chart. What area would you choose as a target for improvement?

### 27.43
Painting new auto bodies is a multistep process. There is an “electrocoat” that resists corrosion, a primer, a color coat, and a gloss coat. A quality study for one paint shop produced this breakdown of the primary problem types for those autos whose paint did not meet the manufacturer’s standards:
<table>
<thead>
<tr>
<th>Problem</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrocoat uneven—redone</td>
<td>4</td>
</tr>
<tr>
<td>Poor adherence of color to primer</td>
<td>5</td>
</tr>
<tr>
<td>Lack of clarity in color</td>
<td>2</td>
</tr>
<tr>
<td>“Orange peel” texture in color coat</td>
<td>32</td>
</tr>
<tr>
<td>“Orange peel” texture in gloss coat</td>
<td>1</td>
</tr>
<tr>
<td>Ripples in color coat</td>
<td>28</td>
</tr>
<tr>
<td>Ripples in gloss coat</td>
<td>4</td>
</tr>
<tr>
<td>Uneven color thickness</td>
<td>19</td>
</tr>
<tr>
<td>Uneven gloss thickness</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Select **Graph ▶ Bar Chart** to make a Pareto chart. Which stage of the painting process should we look at first?

27.48 Table 27.9 in BPS and TA27-09.MTW give process control samples for a study of response times to customer calls arriving at a corporate call center. A sample of 6 calls is recorded each shift for quality improvement purposes. The time from the first ring until a representative answers the call is recorded. Table 27.9 gives data for 50 shifts, 300 calls total. Table 27.9 also gives \( \bar{x} \) and \( s \) for each of the 50 samples.

(a) Select **Stat ▶ Control Charts ▶ Variables Charts ▶ Xbar-S** from the menu to make an \( \bar{x} \) and \( s \) chart. Check for points out of control on the \( s \) chart.

(b) If the \( s \)-type cause responsible is found and removed, then we can remove the points that were out of control on the \( s \) chart. Select **Stat ▶ Control Charts ▶ Variables Charts ▶ Xbar-S** from the menu to make new charts. Find the new control limits and verify that no points \( s \) are now out of control.

(c) Comment on the control (or lack of control) of \( \bar{x} \) of the remaining 46 samples. (Because the distribution of response times is strongly skewed, \( s \) is large and the control limits for \( \bar{x} \) are wide. Control charts based on Normal distributions often work poorly when measurements are strongly skewed.)