How to find the domain of a function?

Let \( y = f(x) \) be a function with an independent variable \( x \) and a dependent variable \( y \).

If a function \( f \) provides a way to successfully produce a single value \( y \) using for that purpose a value for \( x \) then that chosen \( x \)-value is said to belong to the domain of \( f \). If there is a requirement that a \( y \)-value produced by a function must be a real number, the following conditions are commonly checked:

1. Denominators cannot equal 0.
2. Radicands (expressions under a radical symbol) of even roots (square roots, etc) cannot have a negative value.
3. Logarithms can only be taken of positive values.
4. In word problems physical or other real-life restrictions might be imposed, e.g. time is nonnegative, number of items is a nonnegative integer, etc.

**Examples**

\[
f(x) = \frac{1}{x}
\]

(1) applies. Denominator = \( x \neq 0 \).  
Domain = \( \{x \mid x \neq 0\} \)

\[
f(x) = \frac{x}{(x-1)(x+3)}
\]

(1) applies. Denominator = \((x-1)(x+3)\neq 0\).  
Domain = \( \{x \mid x \neq 1 \text{ and } x \neq -3\} \)

\[
f(x) = \frac{x-1}{\sqrt{2}-x}
\]

(1) and (2) apply: \( 2 - x > 0 \).  
Domain = \( \{x \mid x < 2\} = (-\infty, 2) \)

\[
f(x) = \frac{1}{\log(x^2-1)}
\]

(1) applies: \( \log(x^2-1) \neq 0 \) so \( x^2 - 1 \neq 1 \) or \( x = \pm \sqrt{2} \)

(3) applies: \( x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow |x| > 1 \rightarrow x < -1 \text{ or } x > 1 \)

Combining (1) and (3) results in:

Domain = \( (-\infty, -\sqrt{2}) \cup (-\sqrt{2}, -1) \cup (1, \sqrt{2}) \cup (\sqrt{2}, \infty) \)

\[
f(x) = \text{the age of the oldest person in a group of } x \text{ people.}
\]

(4) applies: Domain = \( \{x \text{ is an integer} \mid x > 0\} \)
Domains of selected function types

**Polynomial**
R = All real numbers

**Rational**
All real numbers except zeros of denominator

**Absolute Value**
R = All real numbers

**Root**
R = All real numbers if n is odd OR All nonnegative real numbers if n is even

**Exponential**
R = All real numbers

**Logarithmic**
All positive real numbers

Finding domains from graphs

- **Domain** = \( \{x| -3 \leq x < 4\} = [-3, 4) \)

- **Domain** = \( \{x| x \neq 3\} \)

- **Domain** = \( \{x| x \geq -1\} \)