MATH 2412 Precalculus. Exponential and Logarithmic Functions
Try these problems with someone else in our class. You may work in groups of 2 or 3.

1. On the set of axes below and without a calculator, sketch a graph of the function 
\[ f(x) = 1 + \left( \frac{1}{2} \right)^{x-3} \], labeling all intercepts and asymptotes. Hint: Use the graph of \( y = \left( \frac{1}{2} \right)^x \) and apply translations/transformations. State the domain and range.

2. On the set of axes below and without a calculator, sketch a graph of the function 
\[ g(x) = -\ln(x+2) \], showing all intercepts, asymptotes, and stating the domain and range.
3. A skydiver jumps from a reasonable height above ground. The air resistance she experiences is proportional to her velocity, and the constant of proportionality is 0.2. It can be shown that the downward velocity of the skydiver at time $t$ is given by $v(t) = 80 \left( 1 - e^{-0.2t} \right)$

where $t$ is measured in seconds and $v(t)$ is measured in feet per second.

(a) Find the initial velocity of the skydiver
(b) Find the velocity after 5 seconds and after 10 seconds.
(c) Draw a graph of the velocity function
(d) The maximum velocity of a falling object with wind resistance is called its terminal velocity. From the graph in part (c) find the terminal velocity of this skydiver.

4. Find an exponential function of the form $y = Ce^{kt}$ given that the graph passes through the points (4,6) and (0, 0.5).

5. The number of bacteria $N$ in a culture is given by the model $N(t) = 250e^{kt}$, where $t$ is the time in, with $t = 0$ corresponding to the time when $N = 250$. When $t = 10$ hours, there are 280 bacteria. How long does it take the population to double in size? Sketch a graph of $N(t)$ on the system below or the graph paper provided.
6. Solve the following logarithmic equations.

(a) $4 + \ln 3x = 2$  
(b) $\log_{10} x + \log_{10} (x + 3) = 1$

7. After ten years, a $10,000$ investment grew to $22,568$. Find the rate of interest, $r$, if the money was compounded quarterly during the ten years.

8. How many years will it take for carbon-14 to diminish to 2% of the original amount after the death of a plant or animal? Use the formula $A = A_0 e^{−0.000124t}$. Round answer to the nearest year.

9. A family of functions is given. Draw graphs of the family for $c = 1, 2, 3, \text{ and } 4$. How are the graphs related?

(a) $f(x) = c \log x$  
(b) $f(x) = \log cx$

10. A spectrophotometer measures the concentration of a sample dissolved in water by shining a light through it and recording the amount of light that emerges. In other words, if we know the amount of light that is absorbed, we can calculate the concentration of the sample. For a certain substance the concentration in moles per liter is found by using the formula

$$C = -2500 \ln \left( \frac{I}{I_0} \right)$$

where $I_0$ is the intensity of the incident light and $I$ is the intensity of the light that emerges. Find the concentration of the substance if the intensity $I$ is 70% of $I_0$. 

![Diagram of spectrophotometer]
11. Animal populations are not capable of unrestricted growth because of limited habitat and food supplies. Under such conditions the population follows a *logistic growth model*:

\[ P(t) = \frac{d}{1 + ke^{-ct}} \]

where \( c, d, \) and \( k \) are positive constants. For a certain fish population in a small pond \( d = 1200, k = 11, \) and \( c = 0.2 \) and \( t \) is measured in years. The fish were introduced into the pond at time \( t = 0. \)

(a) How many fish were originally put into the pond?
(b) Find the population after 10, 20, and 30 years.
(c) Evaluate \( P(t) \) for large values of \( t \). What does the population approach as \( t \to \infty \)?
(d) Approximate the value of \( t \) where the rate of change of the population with respect to time is the greatest.

12. If a hot object is put in a room with a temperature \( T_0 \), then according to Newton’s law of cooling, the temperature of the object after time \( t \), in hours, is modeled by \( T(t) = T_0 + Da^t \), where \( 0 < a < 1 \) and \( D \) is the initial temperature difference between the object and the room.

**Problem:** A mathematics professor brings a cup of tea having a temperature of 90°C and places it on a table in a room that has a temperature of 25°C. After 20 minutes the temperature of the tea is 75°C.

(a) Find the temperature of the tea after 1 hour.
(b) When will the tea have cooled to 40°C?
13. In 1935, the American geologist Charles Richter (1900-1984) defined the magnitude $M$ of an earthquake to be

$$M = \log \left( \frac{I}{S} \right)$$

where $I$ is the intensity of an earthquake, measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake, and where $S$ is the intensity of a "standard" earthquake, whose amplitude is 1 micron ($10^{-4}$ cm). Show the magnitude of a standard earthquake is 0. In 1906 an earthquake in San Francisco had an estimated magnitude of 8.3 on the Richter scale. In the same year a powerful earthquake occurred on the Columbia-Ecuador border that was four times as intense. What was the magnitude of the Columbia-Ecuador earthquake on the Richter scale?