THE MAGNITUDE SYSTEM

Anyone who goes outside to see the night sky can tell that the stars are of different brightnesses. It would be nice to have a way to express a star's brightness as a number, rather than just saying "that star's pretty bright." We have devices called **photometers** that can actually count the photons that come to us from the star, but those photons can number in the millions, numbers so large that they become useless.

A system does exist for assigning a (fairly small and easy to remember) number to a star's brightness, and it's been around for quite awhile. About 2200 years ago, the Greek astronomer Hipparchos would take his students out under the stars, and rank them according to their brightness. The brightest stars were called "stars of the first magnitude," the next brightest "stars of the second magnitude," and so on, down to the faintest stars, which were "stars of the sixth magnitude." The original system, therefore, was a very subjective system – that is, a matter of opinion. One person's second magnitude star could be another's third magnitude star, for example. In the modern era, we can assign a magnitude number to a photometer's photon count total, making the system more objective with hard numbers.

The first thing we notice about the magnitude system is that it is "backwards." Brighter stars have lower numbers assigned to them. This is because the magnitude system is a ranking system. If you're a college football team, you want a low number next to your name, preferably number one. Same thing with stellar magnitudes. In the modern system, you can even have negative magnitudes, for the very brightest objects like the Sun and the star Sirius.

APPARENT MAGNITUDE

We were able to keep a lot of Hipparchos' system intact. According to photometer measurements, the faintest stars that can be seen with the unassisted eye are around 100 times fainter than the brightest stars. In the original Hipparchan system, there were 5 magnitudes of difference between these two types of star (6-1). Therefore, the main "rule" of the magnitude system is that a difference of 5 magnitudes corresponds to a factor of 100 times in brightness. If two stars have magnitude values that differ by 5, the star with the lower magnitude is 100 times brighter.

So what if two stars differ by one magnitude? We may be tempted to say that one is twenty times brighter than the other, since 20 + 20 + 20 + 20 + 20 = 100. However, that doesn't work. Magnitudes add together, but brightnesses *multiply* together. So we don't need a number that *adds* together 5 times to make 100; we need a number that *multiplies* together 5 times to make 100. This number is 2.5, or more accurately 2.512: 2.512 x 2.512 x 2.512 x 2.512 x 2.512 = approximately 100. So a second magnitude star is about two and a half times brighter than a third magnitude star. A second magnitude star is not 5 times brighter (2.5 + 2.5) than a fourth magnitude star, but rather 6.25 times brighter (2.5 x 2.5). A second magnitude star is 10 magnitudes different from a 12th magnitude star; this means that the second magnitude star is 10,000 times brighter (100 x 100), not 200 times (100 + 100).

To find how the apparent brightness of two stars compare to each other, you can simply find the difference in magnitudes, and raise the number 2.512 to that power, like so:

$$\frac{B_1}{B_2} = (2.512)^{m_2 - m_1}$$

9 – 2 = 7 and $\frac{B_1}{B_2} = 2.512^7 = 631$, and the 2nd magnitude star is 631 times brighter than the 9th magnitude star.

For another example, we can figure out how many times brighter the Sun is than the brightest star in the night sky, Sirius. Sirius has an apparent magnitude of -1.44 and the Sun has an apparent magnitude of a whopping -26.74. Find the difference and raise 2.512 to that power:

-1.44 – 26.74 = 25.3 and $2.512^{25.3} = 1.32 \times 10^{10}$, and the Sun appears more than 13 BILLION times brighter than Sirius!

ABSOLUTE MAGNITUDE

The original system of magnitudes was developed to measure how bright a star appears to be from Earth, what we now call apparent magnitude. But apparent magnitude is often a poor indicator of how truly bright a star is. For example, the stars of Orion's Belt are some of the hottest, most energetic stars in the Galaxy, and yet they are not among the "top 10" brightest stars in the sky. The reason is that they are so far away. Sirius is the brightest star in the sky, but not just because it is hot and luminous; it is also quite close to us. Distance can make a luminous star look dim and a dim star look deceptively bright.

We can gauge a star's true energy output, called the luminosity, by calculating the absolute magnitude. We remove the effects of distance by mathematically "moving" the star to a standard distance of 10 parsecs from Earth. So the absolute magnitude of a star is the apparent magnitude it would have if it were at a distance of 10 parsecs from us. If we know a star's distance in parsecs (D), and its apparent magnitude (m), we can calculate the absolute magnitude (M) using the formula

$$M = m - 5\log(\frac{D}{10})$$

The “log” symbol is the logarithm function. For our purposes, it’s just a button on your calculator. Let’s do an example with a real star: Procyon in Canis Minor, which is 3.5 pc away, and has an apparent magnitude of +0.40.

$$M = +0.40 - 5\log(\frac{3.5}{10})$$

Do the stuff in parentheses first: $M = +0.40 - 5\log(0.35)$

Now take the logarithm of 0.35: $M = +0.40 - 5(-0.4559)$

Multiply the two negatives to get a positive: $M = +0.40 + 2.28 = +2.70$

Now try it for the famous star Betelgeuse, with a distance of 130 pc and an apparent magnitude of +0.45, similar to Procyon:

$$M = +0.40 - 5\log(\frac{130}{10})$$

Do the stuff in parentheses first: $M = +0.45 - 5\log(13)$

Now take the logarithm of 13: $M = +0.45 - 5(1.1139)$

Note the logarithm was positive this time: $M = +0.45 - 5.57 = -5.12$

Despite the fact that they look almost identically bright to the eye, the two stars have radically different luminosities! Betelgeuse is the more luminous of the two, since its absolute magnitude number is so small as to be negative. How many times more luminous is Betelgeuse? We can use the same technique as we did with apparent luminosity and apparent brightness: find the difference in absolute magnitude, and raise the number 2.512 to that power, like so:

$$\frac{L_1}{L_2} = (2.512)^{M_2 - M_1}$$

2.70 – (-5.12) = +7.82 and $\frac{L_1}{L_2} = 2.512^{7.82} = 1343$, and Betelgeuse is about 1300 times more luminous than Procyon.

If we do this trick with the Sun, which is 0.0000048 parsecs (1 AU) from us, and has an apparent magnitude of -26.74. We can then find out what the Sun's apparent magnitude would be if it were 10 parsecs away:

$$M = -26.74 - 5\log(\frac{0.00000480}{10})$$

Do the stuff in parentheses first: $M = -26.74 - 5\log(0.00000048)$

Now take the logarithm of that small number: $M = -26.74 - 5(-6.3144)$

Note the logarithm was positive this time: $M = -26.74 + 31.57 = +4.83$

And so if the Sun were 10 parsecs from us, it would barely be visible to the naked eye. Earlier we saw that the Sun appears to be 13 billion times brighter than Sirius in the sky. Sirius has an absolute magnitude of +1.5, so it is actually more luminous than the Sun. We can find out how many times:

+4.83 – (+1.5) = 3.33 and $\frac{L_1}{L_2} = 2.512^{3.33} = 22$, and Sirius is about 22 times more luminous than the Sun.

Try it yourself for Procyon (about 7 times more luminous than the Sun) and Betelgeuse (about 9700 times!). Note that we can turn the equation around, and find the apparent magnitude given the absolute magnitude and distance. However, we rarely need to do that, since we can measure the apparent magnitude easily.

ABSOLUTE MAGNITUDE AND LUMINOSITY

We saw above that if we know a star's absolute magnitude is known, we can figure out how its luminosity compares to the Sun. We can turn that around, and figure out a star's absolute magnitude if we somehow know its luminosity. The star's luminosity (L) must be expressed in terms of the luminosity of the Sun; if you only know the luminosity in some other unit (Watts, perhaps), you must divide that number by the Sun's luminosity in the same units. We can essentially turn the process above inside out, and use the formula

$$M = 4.83 - 2.5 \log L$$

Note that the logarithm of 1 is zero, so if the star has the same luminosity as the Sun, it will have the same absolute magnitude, +4.83. Let's do an example with the luminous star Arcturus, which is 110 times more luminous than the Sun:

$$M = 4.83 - 2.5 \log(110) = 4.83 - 2.5\,(2.04) = 4.83 - 5.1 \\ = -0.27$$

Arcturus is rather luminous and fairly close, a combination which makes it the third brightest star in the sky. Now let's look at Barnard's Star, with a luminosity 0.00043 times that of the Sun:

$$M = 4.83 - 2.5 \log(0.00043) = 4.83 - 2.5(-3.37) \\ = 4.83 + 8.42 = +13.2$$

It's a good thing that Barnard's Star is so close. If it were 10 parsecs away, it would be even fainter than it appears from Earth!

You can use the data in the appendix for the brightest stars and the nearest stars to practice some more with these equations.

Updated 1/5/20
By James E. Heath