Objective: The objectives of this laboratory exercise are twofold. The first is to learn how to handle data that has been collected by making repeated measurements in order to reduce measurement error. The second objective is to learn how to Microsoft Excel to produce a graph of your data set and to obtain both a linear regression and the uncertainty in the regression statistics. These are basic skills that you will use the entire semester.

Specifically, you will learn the processes by which the uncertainty in a set of repeated measurements is statistically determined and you will investigate how the uncertainty in a calculated quantity might be obtained.

Introduction: All measurements have an associated inherent uncertainty. This uncertainty is determined in part by the measuring device that was utilized to perform the measurement and in part by the experimenter and the environment. If repeated measurements have been taken then statistical analysis may be used to determine the uncertainty in the measurement process. In this lab you will learn how to use statistics to determine the uncertainty for a repeated measurement. There will be some basic rules associated with the reporting of the uncertainty.

Rules for Uncertainties:
1. The uncertainty should only have one significant digit.
   For example, when the standard deviation of the measurements is calculated you might obtain a value such as 0.03567 cm, this should be reported as the uncertainty in your measurement as 0.04 cm. Since, the first significant digit is the three, but the next digit is a five so then you would round the three up to a four.

2. Always round your reported result
   This will be the average of the repeated measurements, to the same place as the place in which the one significant digit in the uncertainty occurs. For example, if your average were 23.672 cm then you would report your measurement as 23.67 cm, which is rounded to the hundredths place. Otherwise, it appears that your result is more (or less) accurate than indicated by the uncertainty.

3. Both your measurement and your uncertainty must have units and they really should be reported with the same units.
   For example, your final result would be reported to be 23.67 cm ± 0.04 cm or (23.67 ± 0.04) cm. Also if your result is supposed to be reported in scientific notation, then they should both have the same power of ten based upon the result and not the uncertainty. You will move the decimal place the same way in the uncertainty as you did in the result and usually the power of ten is then factored out of the result as a common factor follows: (2.367 ± 0.004) * 10^1 cm.
Exercise #1: Measure the Length and Width of the Front Desk.

Each student will come up and measure both the length and the width of the front desk as precisely as possible. You will use a meter stick as your measuring instrument and you should then record your value for your measurement here:

Length:_______________________    Width:___________________________

After everybody has made their measurements we will place them on the front board to generate a large sample of repeated measurements of the length and width of the front desk. You should record all of the data that was taken from the entire class on a separate piece of paper. You should then count the number of data entries that were obtained and record it here:

Number of data points N:_______

You are going to perform a statistical analysis of the data set that we have just taken. The Worksheet functions in an Excel spreadsheet will be used to carry out the calculation described below

**Average**
The value that best represents the true length and width of the front desk is the mean or average of the data sets. The formula for the average is given by:

\[
\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{x_1 + x_2 + \cdots + x_N}{N}
\]

Where \(\bar{x}\) is the generic symbol for the average quantity and \(x_i\) is the generic symbol for the individual measurements. The subscript \(i\) is an index that stands for the various measurements in any order for 1 to \(N\), the total number of measurements.

**Standard Deviation**
We will then calculate the difference of the measurements from the average by subtracting the \((x_i - \bar{x})\). However this value is not very useful since the average of this quantity is nearly zero every time. So, if we square this value and then take an “average” we will get a use full number call the variance \(\sigma^2\), but we will have to then take the square root to undo the fact that we have squared the values to begin with, this is then called the standard deviation \(\sigma\). There is one technical detail that should be pointed out at this time instead of a standard average we will be dividing by \(N-1\) instead of \(N\), this has to do with the fact that we are working with a sample of a population and not the entire population. The formulas for the variance is as follows:

\[
\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 = \frac{(x_1 - \bar{x})^2 + \cdots + (x_N - \bar{x})^2}{N-1}
\]
The uncertainty in the data set will be taken to be the standard deviation of the data set, which is given by:

\[ \sigma = \sqrt{\sigma^2} \]

The calculations for the length and width of the table are easiest to do in a tabular form.

After you have calculated the mean values and standard deviations of the length and width of the front table, attach your spreadsheet that shows your data set with your calculations and report your results here both the values and ± uncertainties:

Length:__________________________________

Width:___________________________________

Now discuss with the member of your lab group how you would determine the area of the table. What process did you decide to use for the value of the area?

What process did you decide to use for the uncertainty in the area?

What was your final result for the area of the table, both value and uncertainty?

Area:_____________________________________

List several sources of error for this portion of the lab and estimate their effect on your final result. (See the note about human error at the end of the lab!)
Exercise #2: Measurement of the Five Cylinders.

You are going to use an electronic caliper (or a Vernier caliper) and a piece of string and a ruler to measure the diameter and circumference of a set of five cylindrical objects.

<table>
<thead>
<tr>
<th>Cylinder #1</th>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder #2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder #3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder #4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder #5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. You will enter the data that you take into an Excel spreadsheet and produce a graph of the Circumference (Vertical axis) versus the Diameter (Horizontal axis) of the objects.
2. Use the “X-Y Scatter” type of chart.
3. The governing equation should be \( C=\pi D \) but do not force the trendline through the origin. You will need to correctly label the axis and place units on the graph. You should also give the graph an appropriate title.
4. You will then add a linear trendline, which will draw the least-squares best fit line on the graph through your data points.
5. Determine the slope and intercept of the best fit line and their uncertainties. You will need to use the LINEST worksheet function to obtain these uncertainties.

You will turn in your completed data set and graph with this handout.

**Precision**

What is the value of your slope with \( \pm \) uncertainty: ______________

Is the value of \( \pi \) within this range of uncertainty? __________

What is your percent uncertainty? __________

Note: You should only use the value of the slope from the graph where all the data was used to produce one best estimate. Don’t make individual intermediate calculations.

**Accuracy**

You will also need to calculate a percent error in your slope value, since the slope of the line should be \( \pi \). The formula for the percent error is as follows:

\[
\frac{|(slope) - \pi|}{\pi} * 100\%
\]

**Sources of Error:** List several possible sources of error. A listing of “human error” is not an acceptable answer, as this implies that you did the lab incorrectly in which case you should simply redo the lab.