Conservation of Energy Problems

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CHAPTER 6

Given:
\( D = 20.0 \text{ m} \)
\( m = 988 \text{ kg} \)
\( h = 40.0 \text{ m} \)

Questions:
(a) What is \( v \) at top of loop?
(b) Force exerted on car by track at top of loop?
(c) Min height \( h \) to make it over the top?

No friction
No air resistance

(a) \( v \) at bottom of incline is \( v_f \)

\[ mg \Delta h = \frac{1}{2} m v_f^2 \Rightarrow v_f^2 = 2gh \]

At top
\[ \frac{1}{2} m v^2 = mgD + \frac{1}{2} m v_f^2 \]

Multiply by \( \frac{2}{m} \)
\[ 2v^2 = 2gD + v_f^2 \]

\[ v^2 = v_f^2 - 2gD = 2gh - 2gD = 2g(h-D) \]

\[ v = \sqrt{2(9.8)(40-20)} = 19.8 \text{ m/s} \]

(b)
\[ \Sigma F = -N - mg = -mv^2 \]

\[ N = \frac{mv^2}{R} - mg = \frac{m}{\frac{d}{\Delta h}} \cdot 2g(h-D) - mg = \left[ \left( \frac{4(h-D)}{D} \right) - 1 \right] mg \]

\[ N = \left[ \frac{4h - 4D - D}{D} \right] mg = \left[ \frac{4h - 5D}{D} \right] mg = \left[ 4 \frac{h}{D} - 5 \right] mg \]

\[ N = \left[ 4 \left( \frac{40}{20} \right) - 5 \right] mg = 3mg = 3(988)(9.8) = 29.0 \text{ kN} \]

(c) Min height \( h \) \( \Rightarrow N = 0 \)

\[ \frac{4h}{D} - 5 = 0 \]

\[ \frac{4h}{D} = 5 \]

\[ h = \frac{5D}{4} = \frac{5(20)}{4} = 25 \text{ m} \]
CLEARER APPROACH

STAY AT $v^2$ LEVEL

**Bottom**

$v_f^2 = 2gh$

**Top**

\[
\frac{mv^2}{R} - mg = 0
\]

$v_{mn}^2 = gR$

**Min at Top**

\[
v_f^2 = 2gD + v_T^2
\]

$2gh = 2gD + gR$

$2gh = 4gR + gR = 5gR$

$h = \frac{5}{2}R$
What is the total energy of the spring-mass system?

$x = 0$

$E_T = \frac{1}{2} k x^2$

After release:

$E_T = \frac{1}{2} k \Delta x^2 = \frac{1}{2} m v^2 + \frac{1}{2} m g h$

$\Delta x = 0$

$\frac{1}{2} m g h = \frac{1}{2} m v^2$

$\Delta x = \Delta h$

$\frac{1}{2} m g h = \frac{1}{2} m v^2 + \frac{1}{2} m g h$

$\Rightarrow \frac{1}{2} m v^2 = 0$
**Question:** What is the tension in the string at position B?

The mass is traveling in an arc → centripetal motion

\[ \Sigma F = T - mg = m a_c = \frac{m v^2}{l} \]

\[ T = mg + \frac{m v^2}{l} \]

From conservation of total mechanical energy

\[ P_{E_{max \ at \ top}} = KE_{max \ at \ bottom} \]

\[ mg \ell = \frac{1}{2} m v^2 \]

\[ \sqrt{2 g \ell} = v \]
Frictionless

**Question:** Find compression of spring with accel of mass is zero.

**Force on m is** \( mg \sin \theta \)

**Accel = 0 when spring force equals** \( mg \sin \theta \)

\[
SE = kx - mg \sin \theta = 0
\]

\[
x = \frac{mg \sin \theta}{k}
\]

\[
k = \frac{mg \sin \theta}{\Delta x}
\]
CHAPTER 7

E 2. 0.20

A B C D E F K

GIVEN:

\( M = 5.0 \text{ kg} \)
\( v_0 = 20 \text{ m/s} \)
\( \mu_k = 0.40 \)
\( \Delta x = 2 \text{ m} \) (SPRING COMPRESSION)

\\( h = 15 \text{ m} \)

\( \Delta x = 2 \text{ m} \) (SPRING COMPRESSION)

THREE INTERACTIONS: GRAVITY, FRICTION + SPRING.
ALL ARE CONSERVATIVE EXCEPT FRICTION

INITIALLY \( E_T = \frac{1}{2}mv_0^2 \) — REGION A — TOTAL ENERGY FOR PROBLEM

REGION B

\( E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2 + mg \gamma Y; \quad \text{osy} \leq \gamma \)

REGION C

\( E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2 + mgh \)

REGION D

\( E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2 + mgh + fx \); \quad f = \mu_k \gamma = \mu_k mg \)

REGION E

\( E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2 + mgh + fd \)

REGION F (AT FULL COMPRESSION \( V_F = 0 \))

\( E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_F^2 + mgh + fd + \Delta x \)
CHAPTER 7
514 P. 20

Given:
\[ m = 5 \text{ kg} \]
\[ h_0 = 10 \text{ m} \]
\[ k = 200 \text{ N/m} \]
\[ \Delta x = 1.5 \text{ m} \]

Guess: Find \( \Delta E \) lost to sound, thermal, etc.

\[ E_T = mg \frac{h_0}{2} \]

Final:
\[ E_T = mg \frac{h_0}{2} = KE + 6PE + \text{Spring PE} + \Delta E \]
\[ v = 0 \quad \text{so} \quad KE = \frac{1}{2} m v^2 = 0 \]
\[ E_T = mg \frac{h_0}{2} = 0 + mg (5-\Delta x) + \frac{1}{2} k \Delta x^2 + \Delta E \]
\[ 5(9.8)(10) = 5(9.8)(5-1.5) + \frac{1}{2}(200)(1.5)^2 + \Delta E \]
\[ 490 \text{ J} = 171.5 \text{ J} + 225 \text{ J} + \Delta E \]
\[ \Delta E = 490 - 397 \]
\[ \Delta E = 93 \text{ J} \]