MATD 0385. Day 11. Exponential Growth, Day 5. Wed. Feb. 24, 2010

Make-up grade: Test 2 will have between 15-30 points on the material from Test 1. If you work all of them all correctly, and if your Test 2 grade is higher, I will replace your Test 1 grade with your Test 2 grade. If you only get some of them correct, I will have a system where you can earn some extra points on your Test 1 grade. That will be explained later. I hope your goal will be to work them all correctly!

Additional Homework for Module 2:

Rework all of your Test 1 problems. Use multiple sheets of paper so that you can write the problems neatly and clearly and in order, and re-write your corrections on individual problems if you need to as you get feedback. Ask for help from the teacher or a tutor or another student to check your work. Turn this in along with your homework for Module 2 (Exponential Growth.)

We're going to review some of the material from Module 1 (Problem Solving and Logic) as we continue on in Module 2 (Exponential Growth and Decay.)

Review of logic problem: (writing statements in shortened form, writing contrapositive, converse, etc.)

1. Consider the following statement: "If your income is over $2250, then you cannot be claimed by someone else as a dependent."
    a. Choose letters to represent the positive form of each of the individual statements.
    b. Write the original statement in shortened form, using letters and the standard words like "and," "or", "not," "if … then."
    c. Write the contrapositive in shortened form.
    d. Write the contrapositive in a full English sentence.
    e. Do you believe that this contrapositive has to be true if the original statement is true? If not, discuss this in terms of this statement.
    f. Write the converse in shortened form.
    g. Write the converse in a full English sentence.
    h. Do you believe that this converse does not necessarily have to be true if the original statement is true? If so, discuss that in terms of this statement.

Discussion of Negation:

Students missed much more of the material on negating statements than I expected. Clearly we didn't spend enough time on that. Next time, for about ten minutes in class, we we will discuss the material on pages 46-48 of Module 1 about negating statements with quantifiers. Of course, we will also discuss material from Module 2 on Exponential Growth, so bring that book also.

Properties of exponents:

There are several properties of exponents that are very useful to us in our work on these topics. Following are some examples with positive integer exponents and then some definitions to help us interpret negative integer exponents and zero exponents.

1. $2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \;\cdot\; 2 \cdot 2 \cdot 2 \cdot 2 \;=\; 2^7$ (So this is $2^3 \cdot 2^4 = 2^{3+4} \;=\; 2^7$)

2. $\left(2^3\right)^4 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 = 2 \cdot 2 \cdot 2 \,\cdot\, 2 \cdot 2 \cdot 2 \,\cdot\, 2 \cdot 2 \cdot 2 \,\cdot\, 2 \cdot 2 \cdot 2 = 2^{12}$ (So this is $\left(2^3\right)^4 = 2^{3 \cdot 4} \;=\; 2^{12}$)

3. $\dfrac{2^7}{2^3} = \dfrac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$ (So this is $\dfrac{2^7}{2^3} = 2^{7-3} = 2^4$)

4. Motivating negative exponents: $\dfrac{2^7}{2^3} = 2^7 \cdot \dfrac{1}{2^3} = 2^7 \cdot 2^{-3} = 2^{7+(-3)} = 2^4$

5. Thus, a definition: $x^{-n} = \dfrac{1}{x^n}$ for any number x except x = 0. (To avoid zero in the denominator.)

6. $1 = \dfrac{2^4}{2^4} = 2^{4-4} = 2^0$. Thus, a definition: $x^0 = 1$ for any number x except x = 0. (0^0 is not defined.)

There are often multiple ways to express the same relationship. In today's material, we will explore some of these.

Example: If bacterial doubles every five hours, write the formula for its growth in several different forms, including a form which shows the hourly growth rate.

Let n be the number of hours and B be the amount of bacteria at the end of n hours. Usually we would start with a given number of bacteria. Here, however, we're going to assume the initial number of bacteria is 1. That is not very realistic, but it makes it easy for us to focus on the multiplier part of the formula.

$B = 2^{n/5}$ or $B = 2^{0.2n}$

What is the hourly growth rate of this bacteria?

$$B = B = 2^{n/5} = 2^{n \cdot \frac{1}{5}} = 2^{\frac{1}{5} \cdot n} = \left(2^{1/5}\right)^n = (1.148698)^n \doteq (1.15)^n$$

Answer: If bacteria doubles every five hours, the hourly growth factor is approximately 1.15, which means the hourly growth rate is about 15%.

Let's look at the text:

1. Review Lesson 4, the first page, which is page 251.
2. P. 252. Answer the two questions with pencils at the top of this page. For the second question, with the three parts, write three different formulas for each of these, following the example given here.
3. Page 252-253. Activity 1. This leads you through re-discovering the properties of exponents listed at the beginning of this handout.
4. Page 254. Middle. Notice that this is where they introduce the idea about how we can find the annual growth rate from a doubling formula. We did that in the example on the previous page about the bacteria.

Now, let's discuss verifying formulas by tables and by graphing.

Example: Let's make a table of values for the two formulas $B = 2^{n/5}$ and $B = (1.15)^n$. Are these formulas approximately the same?

Here's a table, with some blanks. Fill in the blanks. Notice that these two formulas don't give exactly the same results. Remember that we rounded quite a lot to get the 1.15. What do you think would happen if we used the formula.

n	2^(n/5)	(1.15)^n
0	1.00	1.00
1	1.15	
2		1.32
3	1.52	1.52
4	1.74	1.75
5	2.00	
10	4.00	

(Remember that we rounded quite a lot to get the 1.15. What do you think would happen if we used the formula $B = (1.148698)^n$. We won't do that right now, but you could try it at home.)

Look at Lesson 6, page 273. We have just been talking about writing the same relationship with different bases and using a table to verify the equality. In the table on page 273, plug in 30 to both formulas and find the two values. Do you see what you would do to fill out the whole table to verify this equality?

- Read the top of page 274, where they explain how the rule of 72 helped them figure out what the doubling time was for 3% annual growth.

- Activity 1. Let's all think about 5% interest. What is the formula for A in this case? For the table in number 1, do the three values for x = 0, 5, 10, and 15.

- Now, as a group, discuss page 275, Activity 1, number 2 and start on 3.

Homework: Lesson 6. Pages 274-276. Activities 1 and 2. For Activity 1, you can use any percentage rate you want. I'll write the answer key using 4% annual interest rate.

Review Logic, Lesson 3, Examples 3 and 5 (pages 65 and 66.) From the statement at the end of Example 3, identify some problems in the homework whose solutions are in the back. Do them and check your answers. I do not have any requirement about how many of these to include in your written homework. Do enough that you feel that you understand this.

Then identify some problems like this on your test. Do them and add them to your stack of test corrections. Remember that your stack of test corrections will be attached to your test and placed in your homework notebook when you turn it in next time.

Quiz: 1. Lesson 4, pages 258-259. 9, 10, 11, 12, 13

2. Consider the following statement: "If you can return your iPod touch, then you have not opened the package."
   a. Choose letters to represent the positive form of each of the individual statements.
   b. Write the original statement in shortened form, using letters and the standard words like "and," "or", "not," "if … then."
   c. Write the contrapositive in shortened form.
   d. Write the contrapositive in a full English sentence.
   e. Do you believe that this contrapositive has to be true if the original statement is true? If not, discuss this in terms of this statement.
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