Day 12. Exponential Growth, Day 6. March 1, 2010

Review: Go over negations of quantified statements. See Problem Solving and Logic module, page 47.

Example 1: Here's a statement: "All homes in Austin currently for sale are listed for more than $100,000."

Consider the negation: "Not (all homes in Austin currently for sale are listed for more than $100,000.)"

What would it mean for the negation as stated above to be true? That is, what would you have to show to show that the original statement was not true? Which of these below do you have to show?

There is at least one home in Austin currently for sale for less than $100,000.	All homes in Austin currently for sale are not listed for more than $100,000.

After you think about this, look at page 47, at the bottom. What does it say? Does it make sense?

Can you think of any other ways to write the negation of the original statement that are equivalent to the correct answer? Write them here. Then check to be sure that they are equivalent to the correct negation.

Example 2: Here's a statement: "Some cats have blue eyes."

Consider the negation: "Not (some cats have blue eyes.)"

What would it mean for the negation stated above to be true? That is, what would you have to show to show that the original statement was not true? Which of these below do you have to show?

Some cats do not have blue eyes.	No cats have blue eyes.

After you think about this, look at page 47, at the bottom. What does it say? Does it make sense?

Can you think of any other ways to write the negation of the original statement that are equivalent to the correct answer? Write them here. Then check to be sure that they are equivalent to the correct negation.

Go over quiz answers from Quiz 11.

Go over quiz answers from Quiz 10.

Example 3. Problems like Lesson 5, page 268, problem 4 are quite important because they help you learn about analyzing relationships and formulas. They also give us an example on which to practice your problem-solving skills from the previous module. You wrote a statement in your answer. Now, consider that statement. If it is true, give two examples. If it is false, give one counterexample.

Following up on Lesson 6:

Example 4: Let's find how fast money deposited at 5% annual interest rate will double and write a formula for that in terms of doubling. Let's call A the amount using the 1.05 base and call Y the amount using the doubling formula. Since the rule of 72 gives us a good place to start, we use 72/5, which is about 14 as the place to start. That means the formula is $Y = (1.05)^{n/14}$. To check this, we could make a table just as we did in Lesson 6, activity 1.

n	y=1.05^n	y=2*(n/14)
0	1	1
1	1.05	1.050757
2	1.1025	1.10409
3	1.157625	1.160129
4	1.215506	1.219014
5	1.276282	1.280887
6	1.340096	1.3459
7	1.4071	1.414214
8	1.477455	1.485994
9	1.551328	1.561418
10	1.628895	1.640671
11	1.710339	1.723946
12	1.795856	1.811447
13	1.885649	1.90339
14	1.979932	2
15	2.078928	2.101513
16	2.182875	2.208179
17	2.292018	2.320259
18	2.406619	2.438027
19	2.52695	2.561773
20	2.653298	2.6918

Looking at the table, we see that our doubling formula is going a little bit too fast. It doubles every 14 years, and the 1.05 formula hasn't quite doubled by then.

How should we modify it? We want to make it take a little longer to double. So we can try 14.1 years. Now we could try this out with an entire table. But instead of making the entire table, let's just compute the value at n = 14 to see if it comes closer to matching. $Y = (1.05)^{n/14.1} = 1.990192$
We notice that this is a little bit too large, since the value we're trying to match here is 1.979932.

So we continue make the doubling time a little larger, trying 14.2 and we find $Y = (1.05)^{n/14.2} = 1.98057$

This seems like a good place to stop. We have refined our estimate of the doubling time to 14.2 years.

Notice that, instead of using an entire table each time, we can use use one n-value to check out our guesses. It wasn't essential to use n = 14. We could use any n (except not n = 0. Why not?) If we use a really small value for n, like n = 1, then the differences we're looking for are in the fourth or fifth decimal place, which isn't so easy to look at. So I usually use some value for n near the doubling time or a larger n than that.

Notice that what we just completed can also be thought of as an example we could have used as we thought about Lesson 5, page 268, problem 4. What did we see here? In order to decrease our amount from 1.990192 to our target value of 1.979932, we made the value of the *a* in the denominator of the exponent a little larger. Do you see that? Write down something in your notes about how you see that this is an example of the relationship discussed in Lesson 5, page 268, problem 4.

Now we'll have an introduction to Lesson 7.

Learn about finding the growth factor when the table has unevenly spaced x-values.   In class, begin doing Lesson 7, Activity 1.   Fill out at least two more rows.

Now we'll have an introduction to Lesson 8.   All we'll do of Lesson 8 is page 297-299, to learn what logarithms are.   We will focus on log base 10 for our calculations. Then we'll learn to compute logs on our calculators, so that we can do Lessons 9 and 10.   That will complete our work on this module.

In-class exercise: Since log 1 = 0 and log 10 = 1 and log 100 = 2 and log 1000 = 3 and log 10000 = 4.

Say what integers the following numbers are between.

log 3.2,   log 732,   log 1893,   log 81, log 11, log 9.9, log 9900.

Now compute each of these logs using your calculator and see whether it was between the numbers you expected.

Homework:

~~Lesson 7, Activity 1. Page 290 and 291 Exercises 1 and 2. (part of this is also the quiz.)~~

~~Lesson 7, Homework. Page 293, Exercise 1.~~

Make up two problems like those in Lesson 6, Activity 2, problems 2 and 3. Write the problems carefully and then solve them.   Keep a copy of these for yourself so that you can share them with another person during class next time.   In class, you will solve the problems written by each other.

Quiz:

1. ~~Since log 1 = 0 and log 10 = 1 and log 100 = 2 and log 1000 = 3 and log 10000 = 4.~~

~~Say what integers the following numbers are between.~~

~~log 6532,   log 1.7,   log 93,   log 832, log 8800, log 73, log 2.~~

~~Now compute each of these logs using your calculator and see whether it was between the numbers you expected.~~

2. ~~In Lesson 7, on page 290-291, for the years between 1950 and 1999, give the table, with the values you computed filled in, from exercise 2, give the graph.~~
3. Make up two problems like those in Lesson 6, Activity 2, problems 2 and 3. Write the problems carefully and then solve them.   Keep a copy of these for yourself so that you can share them with another person during class next time.   In class, you will solve the problems written by each other.
4. Give the negation of each of these statements:
    a. Some books are written in French.
    b. All even numbers can be written as a sum of two prime numbers.