Day 13. MATD 0385 Exponential Growth, Day 7   Review and introduction to logarithms.

Test 2: Mon. Mar. 8 – Sun. March 14.   Northridge Testing Center

Review for Test 2.   (This is not a comprehensive review. See the homework for that.)

1. Be able to to do all problems on Test 1.   In logic, pay careful attention to

    a. Write compound statements in shortened form, where you define letters to stand for the positive simple statements.

    b. Write negations of quantified statements. Write negations of "and" and "or" statements using DeMorgan's laws.

    c. Analyze arguments, giving them in shortened form, identifying them as valid or invalid, and identifying them by name. (The same table of names as was given on Test 1 will be given on Test 2.)

    d. This includes writing compound statements in shortened form, including translating appropriate statements to "If .. then" language, where the "if" and "then" are in that order. Overall, it seemed to me that not doing this well hurt students' grades quite a lot.   Of course, this isn't the only thing you need to practice.

2. Be able to do all the types of problems on the Linear Formulas handout.

3. For both linear and exponential relationships, go back and forth between various pairs of verbal descriptions, tables, graphs, and formulas.

4. For data values, recognize that sometimes the same relationship can be modeled by either exponential or linear models. If "slope" is discussed, that's in a linear relationship. If "growth factor", "growth rate", or "percentage growth" is discussed, that's in an exponential relationship. (Lesson 2, pages 233-234, problems 3 and 4. Use differences, ratios, and graphs to illustrate whether each dataset could be modeled in both ways. Then model it as requested in the problem.)

5. Answer questions about doubling. (Lesson 3, pages 244-247 exercises 2, 3, 9 and several other types of questions.)

6. Work with other multipliers, as in tripling, halving, growing by a factor of 5, 10, 1/3, etc.

7. Remember the "rule of 72" for approximating the doubling time for annual percent growth. (It wasn't given, but I'm telling you now about the "rule of 120" for approximating tripling time. You won't have to convert to tripling formulas on this test.   )

8. Go back and forth between a doubling formula and a percentage growth formula.

9. Compute, and recognize when to compute, the geometric mean.

10. Discuss how a general formula depends on the various parts. (Lesson 5, page 268. 3, 4, 5, 6 and Lesson 6, page 275 1abc.)

11. Use some of the analysis of general formulas to refine your work in converting percentage growth formulas to doubling formulas (Lesson 6, Activity 1, pages 274-275.)

New material: Introduction to logarithms.

Many questions involving exponential growth can be answered with logarithms. Also, logarithms are useful in other contexts as well. We will get a quick introduction to logs so that we can explore two specific applications of logs that are useful in science (Lesson 9 on a physics application and Lesson 10 on a biology application.)

In our textbook, Lesson 8 has what we need to know about logs and more. So I'm going to point out just the parts we need.

A logarithm is an exponent. Using logarithms is a way of simply focusing on the exponent in an exponential expression. Look in Lesson 8, pages 297-299.
On page 297-298, for problems 5, 6, and 7, just find the result correct to one decimal place.

Read through page 299, and stop before it starts talking about the natural log (ln). That's more complex than we will do in this course.

Notice that all the bases we use for logarithms are positive numbers, and not equal to 1.

For the rest of the lessons, we will use only the base 10 logarithm. Fill in the following table.

English	Logarithmic Form	Exponential Form
What is the power of 10 that gives 1000?	$x = \log_{10} 1000$	$10^x = 1000$
What is the power of 10 that gives 100000?		
What is the power of 10 that gives 0.01?		
What is the power of 10 that gives 1?		

The log key on your calculator is log base 10.

In-class exercises
1. What two integers is $\log_{10} 73$ between? (Also written as simply log 73.)
2. Use the log key on your calculator to find $\log_{10} 73$.
3. Is it between the integers you chose?
4. Check the value you found in number 2 here, by plugging it into the exponential form and showing that it gives a correct statement.

Homework: Lesson 6, page 280-284. 1, 2, 8, 11, 13, 16, 18

Quiz problems for next class.
1. Work two problems that another student made up for his/her previous quiz. Write each problem, the name of the student who made it up, the solution, and whether that other student agreed that your answer is correct.
2. Use your calculator to find the following: a. log 3824   b. log 0.0238   c. log 1,000,000

Answers to in-class exercises:

English	Logarithmic Form	Exponential Form
What is the power of 10 that gives 100000?	$x = \log_{10} 100000$	$10^x = 100000$
What is the power of 10 that gives 0.01?	$x = \log_{10} 0.01$	$10^x = 0.01$
What is the power of 10 that gives 1?	$x = \log_{10} 1$	$10^x = 1$

Other new material. **Lesson 7.   A Practical Problem: Population Growth.**

1. Uneven intervals of xs. How do you find the annual growth factor?
   Answer: See the beginning of Lesson 7 for discussion.
2. Is there just one formula that to use over the whole domain of x-values?   Or is it OK to have multiple formulas?
   Answer: Yes. See text.   The message – sometimes the growth pattern changes over time, so it is useful to look at some time periods separately.
3. Does a graph of data which has exponential growth ever look linear?
   Answer: Yes. If you look at it over a short enough period of time, even exponential growth looks linear.

Uneven x-intervals.

In an earlier lesson, when we had to find additional y-values for x-values that weren't included in the data, we used the geometric mean.   But here, we aren't looking for the y-values themselves. Instead, we are looking for the growth factor, so that we can write a formula.

So instead of extending the idea of finding the geometric mean of the two y-values, we must start with the ratio of the two y-values and then take an appropriate root.

Consider the world population data. On p. 290, we see some ratios and other values already computed, and then room to compute these for the rest of the table.

To find the first difference and first ratio, we do the same thing as we have done in other tables in this chapter where we found the difference and ratio. (Difference: $0.31 - 0.30 = 0.01$;   Ratio: $\dfrac{0.31}{0.30} = 1.0333$)

However, since we now also need the average annual growth rate (also called average annual growth factor,) we have to take a root of the ratio.

So, in Activity 1, on page 290, for the first of these in the table, we have 1000 years in the interval, so we compute $\sqrt[1000]{\dfrac{0.31}{0.30}} = \left(\dfrac{0.31}{0.30}\right)^{1/1000} = 1.000033$

For the second average annual growth rate (average annual growth factor,) we have 250 years in the interval, so we compute $\sqrt[250]{\dfrac{0.40}{0.31}} = \left(\dfrac{0.40}{0.31}\right)^{1/250} = 1.001020$

If you graph the data in the three time intervals indicated in problem 2 on page 291, you'll see that the first two graphs look exponential and the third graph looks fairly linear.   That's because the third graph has a much smaller time span (1950-1999) and the exponential growth factor is small enough that it would take a much longer time period to see the curve in the graph.

So, it makes sense to model each of these three parts with a somewhat different formula, with the growth factor computed from that average of the growth factors for that time period.

No homework or quiz is assigned over the material in this lesson.

However, if you are interested in answering some of the questions in the exercises, you might find it useful to know data for years after 1999. Here is the world population data I found in www.wikipedia.com.

	Pop'n billions
2000	6.07
2005	6.45
2008	6.71

Nothing on Test 2 on the Lesson 7 material.