Go over quiz problems.

Discuss homework. There are several algebra topics that you are expected to remember here. If you have difficulties with any one of these homework problems, it is important to ask questions and get your questions answered, so that you can do similar problems correctly.

Classwork:

Lesson 3. Pages 23-29. The algebra in this lesson is writing the equation of a line through two points. We have learned to do that in the Linear Formulas handout at the beginning of the semester. Review that carefully at home after class. Practice until you are able to use it correctly. See the end of today’s handout for additional practice problems.

In our Problem-Solving chapter, we learned about Deductive and Inductive reasoning. When I ask you to find the equation of the line between two points, and you use the method we learned in our linear formulas handout, you are using a series of exact mathematical formulas and that is deductive reasoning.

Now let’s look at some data. In Lesson 3, Activity 1, pages 23-25, we have some data about the number of home runs hit by two baseball players during about the first third of the season. We want to use this to predict how many home runs each of them will hit during the entire season of 162 games. That prediction is inductive reasoning. We’ll see how.

First, we can graph Mark McGwire’s results for the first part of the season on a graph that enables us to extend it to 162 games.

Notice that the points are not exactly in a straight line, but they are pretty close to a straight line. There is a way of finding the “best” line to approximate this data, which is called linear regression. You would learn to find that in a statistics course. In this course, I will expect you to just draw the line on a graph and then pick two reasonable points and find the equation of the line between those two points. (Picking which points to use is inductive reasoning.) Often I will tell you which two points to use. For these data, we will use the first and the last points, that is, the point with the smallest x and the largest x. Do you see that, for these data, it is a pretty good approximation to that line given on the graph?

Those are (1,1) and (62,29). First we find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{29 - 1}{62 - 1} = \frac{28}{61} \approx 0.459 \]
Then we use the slope and one of the points on the line to find the y-intercept, b.

\[ y = mx + b \]

\[ 1 = 0.459 \cdot 1 + b \]

\[ 1 = 0.459 + b \]

\[ 1 - 0.459 = b \]

\[ 0.541 = b \]

Then we know that the equation of the line is \( y = 0.459x + 0.541 \).

(After we decided to use the first and last points to write the equation of the line, we have used deductive reasoning for all the steps to here.)

Now, we will use inductive reasoning to say that we expect Mark McGwire will follow approximately the same pattern for the rest of the season, and we will predict his number of home runs after 162 games.

\[ y = 0.459x + 0.541 \]

\[ y = 0.459(162) + 0.541 \]

\[ y = 74.358 + 0.541 \]

\[ y = 74.899 \]

So we predict, based on all the assumptions we have made so far, that Mark McGwire will hit 75 home runs in this season.

Here’s a graph of Sosa’s data.

![Graph](image)

Again, use the first and last points. Can you find these?

Line:

\[ y = 0.328x - 0.64 \]

Prediction for \( x = 162 \)

\[ y = 52 \text{ home runs} \]

Now, in fact, Mark McGwire hit 70 home runs that season and Sammy Sosa hit 66 home runs.

What do you think about these predictions you made? Were they equally good? Do the original graphs suggest that one of them would be likely to be better than the other?

Lesson 3: Do not try to use the method of finding the equation of a line given on pages 26-27. It is correct, but most students find it more confusing than the method we learned in our Linear Formulas handout. It would be an EXCELLENT idea for you to practice some of problems 1-12 at the bottom of page 27 and the top of page 28. The answers to those are at the end of today’s handout.

Lesson 4: Let’s look at pages 36-37. Here they are reminding us of how to look at a graph and pick out the intercept and estimate the slope.
1. On page 36, there are three graphs with the same line on all three. First, what is the intercept?
2. Then remember the slope as rise/run and find a couple of points and estimate the slope. Notice how they did it.
3. Then put these together into the equation of the line.
4. Sometimes the intercept is not actually shown on the graph and you have to estimate it.
5. Sometimes it is hard to see exactly what the coordinates of a point are and so you have to estimate them. If you do it differently than someone else, you’ll get a slightly different answer for the slope.
6. Now, on page 37, here in class, do problems 1, 2, 4, and 6.
7. When I have taught this in previous semesters, mostly students didn’t actually estimate, but just found two points and computed the slope and intercept. That defeats the purpose here, which is to be able to look at the graph and see approximately what the answers are, so that you can “trouble-shoot” and see whether the answer you computed is a reasonable answer.

Homework:
Lesson 2: page 21. Problems 10 and 11. (Answers at the end of this page.)
Lesson 3: Do page 29, 5 abc. (Answers in the back of the book.)

Quiz:
Lesson 3, pages 28-29. 1, 2, and then two more problems, labeled 4A and 4B below:
Labeling the variables: Since we learned to find the equation of a line using \( x \) and \( y \), we will rename the two variables \( x \) and \( y \). Since we are predicting the cost per fish on the graph that was requested, we will make the cost per fish the \( y \)-variable, and so the number of fish is the \( x \)-variable.
4A. Find the equation of a line through the data here using the first and last points, that is, using the point with the smallest \( x \) and the point with the largest \( x \).
4B. Now find the equation of a line through the data using the first two points, that is, the two points in the dataset with the two smallest values of \( x \).
Comment: I can see, just by looking at the graph and making a quick sketch of the lines, that the line through the first two points is steeper and has a higher intercept than the line through the first and last points. Can you see that must be true from looking at your graph?

Answers to Lesson 2, page 21: 10. \( E = y \) and \( R = 66x \), so \( E = R \) means \( y = 66x \)
Notice that $66 is the slope of the revenue line.
11. \( E = y \) and \( R = mx \), so \( E = R \) means \( y = mx \)
Notice that \( m \) is the slope of the revenue line. This is the formula for a line with \( y \)-intercept of 0.

Answers to the “find the equation of the line” problems on pages 27-28.
1. \( y = -3x + 11 \)  2. \( y = -2.5x + 5 \)  3. \( y = 1.5x - 2 \)  4. \( y = 0.6x + 4.8 \)
5. \( y = -0.05625x + 8.625 \)  6. \( y = -\frac{1}{7}x + 5 \)  7. \( y = -\frac{9}{7}x + 5 \)  8. \( y = \frac{4}{7}x + \frac{18}{7} \)
9. \( y = -\frac{1}{80}x + 87.5 \)  10. \( y = 28x \)  11. \( y = 0.75x + 5 \)  12. \( y = -0.05x + 7 \)