Logic is a way to find what you actually already know, but you may not be aware of it.

That is, sometimes the things you already know imply that something else has to be true. But you may not yet have thought of that something else. Logic helps you see that quickly and notice that it follows from what you already know.

For examples of this, see Lesson 4, exercises 8 and 9 (page 77). In these problems, we assume that you know the two things above the line. The question is whether those imply the statement below the line. If so, we say that "the argument is valid." Look at 8 and 9. Do you see now that one of those arguments is valid and one isn't? Which is which and why?

So it is also the case that we can use logic to avoid some mistakes that people sometimes make about what they think they know, but that they don't actually know.

I don't expect you to be good at this yet – I just want you to see what the point is about what we are doing here.

What sorts of statements are appropriate for us to use in logical arguments?

Sometimes people get annoyed with / don't believe logic because they feel that they are being told they must agree to something that they don't agree with. Sometimes that is because of some missing assumptions, or missing information.

What is a statement? Our text says "A simple statement is a declarative sentence about one idea that is either true or false. We don't have to know which it is – true or false."

An example of a declarative sentence that I don't consider to be a simple statement: "The presence of television in a household is good for children." My problem with this is that "good for children" is very "squishy." If two people get into a conversation about this, the only productive thing they can do is to talk about some of the things about television that they think are positive influences and other things that are negative influences and then they can discuss the importance of each of those things in their own thinking. (Maybe some individuals can only think of negative influences but even they should admit that not everyone can only think of negative influences, and vice versa.) So this short declarative sentence is masquerading as one idea, but it is actually summarizing several ideas based on some assumptions about the meaning of "good" and those assumptions aren't clearly stated.

Another reason this sentence is rather "squishy" is that "the presence of television" isn't very clear. Does that mean a television the children can watch broadcast TV on? Or does it include a television that can only be used with videos or DVDs? So there's some relevant information that is missing.

Summary: For logic to give us a good conclusion, it relies on the statements included being "crisp" statements. And "crisp" statements are statements where the information needed to define the terms and the assumptions needed to understand them are clear to the people participating in the conversation.

The meaning of "and" and "or":

Meaning of "and" Easy. Both of the statements are true.

Meaning of "or": Not so clear. Does it mean that only one of the statements is true or that at least one of the statements is true?

Example 1. Here are some statements from real life that are pretty clear.

- You will have homework in your marketing class on either Monday or Wednesday.
- You will have homework in your marketing class on Monday or Wednesday, or possibly both.

To have word to describe these, we could say "exclusive or" and "inclusive or". Which is which?

In real life, if your teacher says "You will have homework in class on Monday or Wednesday" which of those two statements above does she mean? Is it clear? What would you understand it to mean?

So we might say that "or" can mean two different things and which one depends on the context. That's certainly not very satisfactory for a mathematical logic system. So, in mathematical logic, they had to choose one of these for the meaning of "or." They chose the "inclusive or." So, in mathematics, "or" means one or the other or possibly both.

Why? Well, many things in the system are easier and simpler when we use the inclusive or. We'll talk about that more when we get into negatives.

Quantifiers: "All" and "some (there exists, there is at least one)" are quantifiers.

Negatives of quantifiers:

All: In everyday language, for the negative, people might say "All are not" or "not all are". (Do you see the difference in meaning?) In logic, we mean the second of these. That's because we think of the negative of “all” as what exactly must be true to say that “all” is not true. So that is “not all are”, which can also be thought of as “at least one is not.”

Some: In everyday language, I think it isn’t clear at all what people mean by “not some.” Maybe they mean “some are not” or “all are” or “none are.” Very “squishy!” In logic, we mean by the negative, we mean exactly what it takes to make “some” false. And so that must be “none are,” which has the same meaning as “all are not.”

Summary of Quantifiers and their negatives:

- The negative of “All” is “at least one is not.”
- The negative of “Some are” is “none are.”

More discussion of the “inclusive or” and negations.

Suppose we have this compound statement: Janet is tall and Janet is 15 years old.
How can this be false? There are three ways:
(1) Janet is not tall and Janet is 15 years old.
(2) Janet is tall and Janet is not 15 years old.
(3) Janet is not tall and Janet is not 15 years old.
We can summarize these into one statement: Janet is not tall or Janet is not 15 years old, or both.

If we write this statement as “Janet is not tall or Janet is not 15 years old” and interpret the “or” as the “inclusive or” then this statement is the negation of the original compound statement.

Negation of a statement: Another statement which is the minimal change needed to be false exactly when the original statement is true.

Original statement	**Negation of the statement**
“A and B”	“not(A and B)” is the same as “not A or not B.”
“A or B”	“not(A or B)” is the same as “not A and not B.”
“All are.”	“not (All)” is the same as “at least one is not.”
“Some are.”	“not(Some are)” is the same as “none are.”
“not A”	“not(not A)” is the same as “A”

Section 1 has many definitions and symbols. We’ll see the important ones as we do some examples. It is important that you learn to read the material here and know the meanings of the symbols. It is less important that you use the symbols yourself. You must learn to summarize statements in brief notation, but you can use some words.

Sentence	**Definitions of the simple statements**	**In symbols**	**In brief notation**
Mr. Lowe is not a high school graduate	G= Mr. Lowe is a high school graduate	~ G	not G
The bag is blue and large.	B = bag is blue L = bag is large	$B \wedge L$	B and L
The shirt is red or sleeveless.	R = the shirt is red S = the shirt is sleeveless	$R \vee S$	R or S
If I go to a movie, then I eat popcorn.	M = I go to a movie P = I eat popcorn	$M \rightarrow P$	If M then P
I will eat if and only if I am hungry.	E = I will eat. H = I am hungry.	$E \leftrightarrow H$	E if and only if H
I will go to bed, but I am not tired.	B = I will go to bed. T = I am tired.	$B \wedge \sim T$	B and not T

Simple statements are positive. In these definitions, when a statement is clearly a negation, make your letter stand for the positive statement, which is the simple statement, and then use a negation symbol to show the given compound statement, as in the first and last examples in the table above.

Other connectives: Sometimes we can translate other English connector words to logical connectives. Notice that the word "but" is not one of the basic logical connectives. In logic its closest equivalent is "and." It is interesting to think of the nuances we mean when we say "but" instead of "and." In logic, however, we simply translate it as "and."

Material from Section 2.

Truth Tables. The systematic way that mathematicians investigate whether statements are logically equivalent is with truth tables.

Example 3: **We are going to use truth tables to investigate how to write the negation of the statement "a and b" in order to see how it relates to "or."**

Here's a truth table for the statement "a and b".

a	**b**	**a and b**
T	T	T
T	F	F
F	T	F
F	F	F

Notice that in the first two columns, we put a **systematic** listing of **all** the possibilities for whether statements a and b are true or false.

In the third column, we list whether the compound statement "a and b" is true or false for the given values of the truth of the simple statements.

A negation of a statement means that True and False are reversed. So this is a truth table for the negation of the statement "a and b."

a	**b**	**a and b**	**~(a and b)**
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Now, earlier I said that we use the "inclusive or" in math for several reasons, and one of those has to do with the negation of "a and b."

Let's investigate the statement "not a or not b" where we understand this to be the inclusive or.

a	**b**	**~a**	**~b**	**~a or ~b**
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Notice that, if we had used the "exclusive or" then the last line of the table would have given us a False and so it would not have matched the negation of the compound statement "a and b."

Example 4: Now we will investigate the negation of the compound statement "a or b".

a	**b**	**a or b**	**~(a or b)**
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Now let's look at the statement "not a and not b"

a	**b**	**~a**	**~b**	**~a and ~b**
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

So we can see from these truth tables that the negative of the statement "a or b" is logically equivalent to "not a and not b"

Examples 3 and 4 are summarized as "DeMorgan's Laws" in Section 2, on page 57.

Example 5: What does the double negative mean?

In logic, where any statement is either true or false, then "not true" must mean false. So then "not false" must mean true.

a	**not a**	**not (not a)**
T	F	T
F	T	F

So "not (not a)" is logically equivalent to "a".

I WILL NOT ask you to make truth tables. In the text, they make truth tables somewhat differently than I did here. (I prefer the method they describe in Example 8.) The point of truth tables is to use symbols to clarify the meaning. I suspect that most students in this class find it more useful and easier to understand if we discuss the meaning rather than how to manipulate the symbols.

Also, in the text, they are working up to looking at more complex statements than we will do. Truth tables are particularly useful for dissecting complex statements such as those in problems 28-34 and 52-55. **I want you to learn the basic ideas and deal with the less complex statements.**

Section 1, pages 48-50:

Class: 2, 5, 6, 10, 11, 12, 22, 24, 28, 30, 42

Homework: 15, 17, 19, 21, 25, 29, 31, 33, 41, 43, 45, 47, 49, 51, 53

Section 2. Pages 59-61 (Notice that the book has answers to the even-numbered problems in this section.)

Class: 37, 41, 43, 48, 63

HW: 36, 38, 39, 40, 42, 44, 46, 48, 62, 64

Quiz:

1. Page 49. problem 27
2. Page 49. problem 46
3. Page 60, problem 45
4. Page 60, problem 47.

5. A cellular phone company has equipment that can service 80 thousand customers. In 2000 they had 57 thousand customers and, over the last few years, they have been adding about 3 thousand customers per year.
Let x stand for the number of years since 2000 and y stand for the number of customers, measured in thousands (so that y = 60 means 60 thousand customers.)
a. Make a table of the number of customers in each of the years corresponding to $x = 0, 1, 2, 3$, etc. through 7.
b. Graph this.
c. Use the pattern to write a formula for y in terms of x.
d. Interpret the slope in terms of the variables in the problem.
e. Interpret the intercept in terms of the variables in the problem.