What does logic tell us about conditional statements?

When I surveyed the class a couple of days ago, many students said that when someone tells you an "if .. then" statement you just think of it as the two things being related than instead of thinking it had a specific direction to the relationship.   And I understand that.   In common language, we can usually tell from the context whether there is a direction or not.   For example, when you hear "If it is raining when I leave home, then I will carry an umbrella," you think of associating raining and carrying an umbrella.   Many of us will say that exact statement in day-to-day life, and it is true, and moreover, it is also true that if we are brought an umbrella when we left home, then it was raining, because we only bring out that umbrella when it rains.   So many of us mean that to be a two-directional statement, even if we use the "if, then" words.   This makes it confusing to come to a math, science, or social science class where we are supposed to use more precise language.

In more precise language, we have both **conditional statements and bi-conditional statements.**

Conditional: If a person is teaching college-level math at a Texas community college, then that person has earned 18 graduate hours in mathematics.

Bi-conditional: An undergraduate student at Smith University earns a minor in mathematics if and only if he earns credit in 12 hours of upper-division mathematics.

What is the difference between the structure of a conditional statement and a bi-conditional statement?

Do you think that the conditional statement above is also true if you swap the two parts? (Answer: It is not.)   Do you think the bi-conditional statement above is true if you swap the two parts? (Answer: It is.)

Now we're going to spend some time looking at the conditional, which has one specific direction. The main form of a conditional statement is "if p, then q."   Here p is called the hypothesis and q is called the conclusion.

First we will discuss different ways to say this relationship in an English sentence.   Then we will discuss other, related, conditional statements and whether they are equivalent to the original conditional statement or not.

Common language	Rewritten in "If .. then"	Explanation
I will go to the movie if you pay for it. **G if P** G = go to the movie P = you pay for it	If you pay for it, then I will go to the movie. **If P, then G**.	The hypothesis is what follows the "if" in the sentence. "If" tells us what is sufficient for the other thing to happen.
Mrs. Hernandez* will move to a nursing home only if she becomes unable to walk. **M only if U** M = move to nursing home U = unable to walk	If Mrs. Hernandez* moves to a nursing home, then she became unable to walk. **If M, then U**.	"Only if" in the middle of the sentence gives the opposite direction from "if" in the middle of the sentence.
Your paying for it is necessary in order for me to go to the movie. **P is necessary for G**. G = go to the movie P = you pay for it	If I went to the movie, then you paid for it. **If G, then P**.	Since it is necessary, then if the other thing happens, that implies that the necessary thing happens. The "necessary" thing is the conclusion.
For Mrs. Hernandez*, her moving to a nursing home is sufficient to tell us that she has become unable to walk. **M is sufficient for U** M = move to nursing home U = unable to walk	For Mrs. Hernandez*, if she moves to a nursing home, then she has become unable to walk. **If M, then U**.	Since it is sufficient, if it happens, then the other has to happen. The "sufficient" thing is the hypothesis.
I get scared when I go to a horror movie. **S when H** S = scared H = go to horror movie	If I go to a horror movie, then I get scared. **If H, then S**.	What comes after "when" is what it takes to make the other thing happen. What comes after "when" is the hypothesis.

* Mrs. Hernandez is a friend of mine who is 89 years old, owns her three-bedroom home, has several family members who help her, and enough money to pay some paid care-givers. But she is fairly large, and one person cannot lift her.

Main result: When we have a **conditional statement "if p, then q"** then

- **The converse, "if q, then p" may or may not be true.** The statement "if p, then q" gives us no information about whether the converse is true.

- T**he contrapositive, "if not q, then not p" must be true.**   This is logically equivalent to the statement "if p, then q."

- **The inverse, "if not p, then not q"** is logically equivalent to the converse, so it **may or may not be true**.

I know that our textbook does a lot with truth tables here, but I find that less than satisfactory for several reasons.   I think it is good to just think about these and do some examples, until you understand WHY this result is true: **A conditional statement is logically equivalent to its contrapositive.**

Exercise: Here is a conditional statement that is true about me.

"If I ride on a roller coaster, I get dizzy."

Write the converse.

Write the contrapositive.

Write the inverse.

Look at the meanings here.   Think about the fact that I also get dizzy when I close my eyes and whirl around in a circle several times.   Which of those three statements is logically equivalent to the original statement? (Hint: There is only one of these that is equivalent.)

Let's think about the bi-conditional statement about math majors at Smith U. We'll break it up into the two conditional statements, and then write the converse, contrapositive, and inverse of each of the two conditional statements.

An undergraduate student at Smith University earns a minor in mathematics if and only if he earns credit in 12 hours of upper-division mathematics. **M if and only if C.** M = earns a minor in math C = earns credit in 12 hours of upper-division math.		
If an undergraduate student at Smith University earns a minor in mathematics then he earns credit in 12 hours of upper-division mathematics **If M, then C. TRUE**		If an undergraduate student earns credit in 12 hours of upper-division mathematics then he earns an minor in mathematics. **If C, then M. TRUE**
Converse: If C, then M.		Converse: If M, then C.
Contrapositive: **If not C, then not M**. **TRUE**		Contrapositive: **If not M, then not C**. **TRUE**
Inverse: If not M, then not C.		Inverse: If not C, then not M.

Each of the statements in bold in that table above must be true, because of what we have learned before. (That the contrapositive of a true statement must also be true.) For a single conditional statement, we know that the converse and the inverse may or may not be true.   So those aren't in bold here. HOWEVER, draw some arrows to indicate how each of those is the same statement as one of the statements in bold.   When you have finished that, **you see that, for a bi-conditional statement, all of these statements are true**.

How to mislead people: Make a conditional statement, then treat it as if you made a bi-conditional statement.

A vote for the Green Party means you are concerned about environmental issues.
A vote for the Libertarian Party means you are concerned about personal freedom.
A vote for the Republican Party means you are concerned about high taxes.
A vote for the Democratic Party means you are concerned about preventing racial discrimination.

Each of these is "If you vote for ___, then you are concerned about ____"
But we can all see discussions sometimes that try to get you to believe "If you are concerned about ___, then you will vote for the ___ party."

In real-world language, sometimes people do mean "if and only if" when they say "if, then."
But I caution you to avoid that and be clear when you mean that the implication goes both ways.

Consider Section 3, number 46.
Must you read the first sentence in problem 46 as "if and only if" even though it doesn't say
that? Answer: No. It is possible that someone might say that sentence when they mean if and
only if, but, then again, they might mean exactly what they say – a one-directional conditional.
Consider the following example.

Example: On the registration site for my math conference, which is Nov. 7-11, it says
Jul 1-Oct. 15, registration fee: $320. Oct. 16 – Nov. 10: registration fee: $320 + $40 late fee =
$360.   So here are statements.

   a.   If you pay after Oct. 31, you must pay the late fee.
   b.   If you pay after Sept. 1, you must pay the late fee.
   c.   If you pay after Oct. 20, you must pay the late fee.
   d.   If you pay after Oct. 15, you must pay the late fee.
   e.   You must pay the late fee if and only if you register after Oct. 15.

   Write True or False, as appropriate, for each of these statements a. – e.   (Write it beside each
   statement.)

   Which one of these statements (a through e) is the most useful statement for a person
   registering for the math conference?

   Which of the other statements is one that might be made instead of the most useful
   statement?

   Suppose my school won't approve my travel request until Oct. 20. Which of these
   statements is the most relevant to me?

Section 4.

It is not necessary for you to use the exact language that the book uses for their identification of arguments.

It IS necessary for you to be able to identify which of the arguments is being used, by some name or description, and whether it is valid or invalid.

Valid Arguments and Invalid Arguments

Read the four forms of valid arguments and the two forms of invalid arguments on page 74.   Take notes that will help you remember what these mean. (Either copy the symbols or make up an example, or both.)

Valid: Law of Detachment

Valid: Law of Contrapositive

Valid: Law of Syllogism **(Chain)**

Valid: Disjoint Syllogism (**Meaning of "or"**)

INVALID: Fallacy of the Converse

INVALID: Fallacy of the Inverse

Are these the only forms of arguments?

No. For one thing, some arguments are more complex and have three lines of premises instead of only two. There are some arguments with only two lines of premises that we don't discuss in this book.   But the six arguments discussed here are the most common types of arguments.

How to logicians decide if a new type of argument is valid?

They make a truth table for it. In our textbook, Example 1 proves the Law of Contraposition using a truth table and Example 2 shows the Fallacy of the Inverse using a truth table.

What are some of the difficulties in using logic in real-life situations?

Difficulty 1. Sometimes people don't distinguish between the truth or falsity of the individual statements and the validity of the argument.

Logic problems in math classes: Sometimes some problems are inserted with clearly incorrect simple statements, but in a **valid argument form**.   So you must be able to identify it as a valid argument form.   It is also OK to comment on whether an simple statement is true or false.   But when you say that an simple statement is true or false, that DOES NOT address the question about whether the argument form is valid.

Difficulty 2. Sometimes there is other information besides the information given in the argument that seems relevant to you.

Logic problems in math classes: Here we are practicing analyzing arguments and you are expected to not bring in additional information, but only use what is given in the problem.   We math teachers hope that, in real life, you will be able to use these tools to decide which parts of your new information can be brought in in ways that enable you to use logic with that information as well.   Of course, in real life, when you are making decisions, it is always possible that some relevant information cannot be brought in in this way. So then you may need to decide whether your situation is or is not appropriate for using pure logical arguments to arrive at a conclusion.

Difficulty 3. A statement could reasonably correspond to different logical structures (for example, a conditional or a bi-conditional.)   In this case, you should recognize that those will lead to somewhat different logical conclusions.   So do one of these:

- Don't trust the statement.

- Ask for more information.

- If you're the one writing it, then clarify it.

- If you're reading it, and can't get more information, then state clearly how you are interpreting the statement before using the statement to reach a conclusion.   (Employers will be very impressed with this!)

 From the text:

Section 3: pages 68-70
Class: 32, 28, 46, 48
HW: 27, 29, 31, 33, 45, 47, 49, 51, 53, 55, 57, 59

Section 4: pages 76-79
Class: 8, 10
HW: 17-23 all odd-numbered problems.

Quiz: Pages 68-70. 50, 56, 60.   Pages 76-79: 12, 16, 18