MATD 0385. Day 8, Feb. 15, 2010 Exponential Growth. Lessons 2 and beginning Lesson 3

Lesson 2 and beginning Lesson 3: Topics

1. Understand the structure of the formula for linear growth. (Discussed in the handout at the beginning of the semester.)
2. Understand the structure of the formula for exponential growth. Discuss how it is similar and how it is different from the formula for linear growth.
3. Understand that each of these relationships can be summarized in four different ways (words, table, graph, and formula).
4. From a table of values where the x-values begin with 0 and increase in increments of 1, if the data fit either a linear formula or an exponential formula, write that formula.
    a. Write that formula with x and y.
    b. Write that formula with other variables, like *n* and *C*.
    c. Write that formula in function notation, as the book does in Lesson 2.
5. Go back and forth between the four different representations of a relationship.
6. Recognize that real data which can be well-described by one of these forms of growth doesn't necessarily have exactly equal differences or exactly equal ratios.
7. Recognize that, if the exponential growth is gradual enough (that is, with a growth factor close to 1.0000) then from a table of values, it is often hard to distinguish whether it is better summarized by a linear formula or an exponential formula. Sometimes we will try to model the relationship in both ways.
8. Write a formula for a dataset where the y-values double for each increase of 1 in x.
9. Understand why this is easier if we call the beginning x-value zero instead of 1.

While working the problems from the quiz from last time, I talked about the following:

Deductive Reasoning: The examples I provided in the previous class for exponential and linear growth were not real-world examples. The data were constructed mathematically by using a formula. I asked you to look at the data, find the differences and/or ratios, and, using that, tell me whether the formula was linear growth (or decrease) or exponential growth (or decay.) So when we talked about the differences being constant or the ratios being constant, they really were exactly constant, because the numbers were constructed from a mathematical formula.

Inductive Reasoning: When we have real-world data, even if the pattern is very linear or the pattern is very exponential, it will not usually have exactly constant differences or ratios. There is usually round-off error and measurement error involved in obtaining the y-values. And the process itself may have some "noise" in it, which means the actual values vary somewhat from those a formula predicts. So, we are just looking for whether the differences seem about the same or the ratios seem about the same. Do you see that this is inductive reasoning as we discussed it in Module 1?

When we go to the college-level math courses such as 1332, 1333, or 1342, it is important to be able to extend our ideas from the exact numbers and relationships we get from mathematical formulas to the approximate numbers we obtain from measurements and the relationships that are approximately linear or exponential.

Here's a very famous statement about this: "No models are correct, but some models are useful." This was said in the context of real-world applications. We will think about this as we go through these modeling exercises.

Starting Lesson 1:

At the beginning of Lesson 1, we see an example of a linear relationship. We studied this earlier in the semester on that special handout. Do you recognize this from then? The notation is somewhat different.

Different ways of writing formulas

We are used to writing formulas with x and y. But in that notation, if we want to emphasize what the x-value is that produced a y-value, we can only do that with words. We can say "if x is 3, then y is 7." Mathematicians prefer to have a more concise form for this statement, so they developed function notation. We would write $y(3) = 7$ and say "y of 3 is equal to 7." **Notice that the parentheses here DO NOT mean to multiply anything.** They are simply a way of keeping track of the x-value in the middle of a statement about the y-value.

Read the examples at the beginning of Lesson 2 which use this notation. We often use this notation with "meaningful" variables, which are variables whose letter suggests what the variable is. In the first example we have the variables p and n, where p is population and n is the number of years.

I will not require you to use this notation, which is called function notation. I will require you to be able to read it correctly. In particular, you must know that p(n) in this context DOES NOT mean to multiply p and n.

The second and third pages of Lesson 1.

Here we discuss the four different ways we can describe a relationship (words, formula, table, graph.) And we see that we can write these formulas from the tables for exponential growth as well as linear growth, in a similar way.

Write the formulas for the three Datasets at the beginning of the handout for Lesson 1 of Exponential Growth.

A: B: C:

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When we are writing the formula, does it matter whether we start with x = 1 or x = 0? What is the difference? Is one of these easier than the other? Consider these two examples, write the formulas, and say which is easier.

Example 1

n	C	C
1	3	3
2	6	3*2
3	12	3*2*2
4	24	3*2*2*2
5	48	3*2*2*2*2
6	96	3*2*2*2*2*2

$C = 3(2)^{n-1}$

Notice that the exponent is a bit confusing here. That's because we started with n = 1.

Example 2

t	A	A
0	11	11
1	22	11*2
2	44	11*2*2
3	88	11*2*2*2
4	176	11*2*2*2*2
5	352	11*2*2*2*2*2

$A = 11(2)^t$

Notice that the exponent is easier here. That's because we started with t = 0.

Usually, if we have a choice, we'll start with the input variable as zero instead of 1. That makes the resulting formula easier.

We will skip Lesson 2, Activity 1 for now. The data are far enough from linear and exponential that it is rather hard to decide which to use. In the homework for this section, the data is more clearly linear or exponential and so that is a better place to start looking at making a model to approximate real-world data.

We don't really need to cover the section on Nitty Gritty of Linear Functions because we have already studied these in the handout earlier in the semester.

Lesson 3: Discuss Doubling. In this lesson, when we double, we use a common ratio of 2. This lesson also discusses the difference between starting at x = 0 and starting at x = 1, which we already discussed. If, instead of doubling, we divide in half each time, we can do all the parts of that problem and see what we get. That's Activity 1 in this lesson. We will do it.

Classwork:

Do p. 233-237: problems 3 and 9
While discussing problem 3, we discuss changing from 1955 to the variable, x = years since 1955. And then we change from x = years since 1955 to n = number of five-year increments since 1955. When we do this, we get a table where the input variable starts at 0 and goes up by 1 each time.

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While discussing problem 9, we talk again about the relationship between the ratio, which we also call the growth factor, and the percent increase. We first discussed this relationship while solving Lesson 1, problem 6, which was a quiz problem from last time.

Lesson 3: p. 241. Activity 1, problem 1.

Here we notice that the common ratio, the growth factor, can be less than 1, and so the graph is decreasing. This is exponential decay instead of exponential growth.

Homework:

Lesson 2: p. 233-237: 1, 3, 5, 6, 9, 11
Lesson 3: p. 241. Activity 1.
Lesson 3: p. 245 problem 5

Quiz problems:

Lesson 2, page 233-237: 2, 4, 7, 8