Initiating Lesson 3.1.1: Introduction to Scatterplots and Bivariate Relationships

Estimated number of 50-minute class sessions: 2

Learning Goals

S.2. Distributional Thinking Goal: Students will demonstrate the use of distributional thinking to reason about the data in order to describe and summarize distributions of data, identify trends and patterns, judge the fit of a model to a distribution, and describe similarities and differences in comparing distributions.

The series of tasks in this introductory lesson are designed to motivate an initial and informal understanding of concepts related to interpreting scatterplots.

Specifically, you want students to gain a preliminary and informal *understanding* that

- each point on the scatterplot represents a single observation consisting of measurements on two variables.
- an overall downward trend in the data indicates that small values for x tend to correspond to large values for y. Larger values for x tend to correspond to smaller values for y. Also with a similar understanding of overall upward trends.
- the accuracy in a prediction is related to the variability (scatter) in the data. Variability can be explained by increases in x or by additional factors having influence on y.

Students should begin to learn *how* to

- interpret the meaning of particular points on the scatterplot.
- without the benefit of formal development of the statistical concepts of association, correlation, and regression, recognize directional trends in the distribution of bivariate data and use these trends to make predictions.
- assess the strength of the relationship informally by looking at the degree of scatter.
- develop plausible explanations for the variability seen in the data.

In this series of tasks, you are not formally developing the statistical concepts of correlation or regression. Rather, you are working to build students' ability to see associative trends through the noise of real data and to make decisions in the face of variability. After this lesson, students are introduced more formally to the following concepts:

- form of a relationship as linear or nonlinear,
- correlation coefficient as a measure of the strength of a linear association, and
- least squares regression line as a way of describing central tendency of a bivariate distribution (much like the mean describes the central tendency of a univariate distribution).

Developmental Math Connections

Since this is an introduction to working with bivariate data, the learning goals are focused on interpreting scatterplots and motivating an initial and informal understanding of concepts related to distributional thinking with bivariate data that underlies modeling data with a linear function. In future lessons, these ideas become more explicit.

You might wonder why the authors did not begin with a discussion of the Cartesian plane or an exercise in which students construct a scatterplot, as you might in an elementary algebra course. The rationale is

multifold. First, in Statway, you have a more heterogeneous group of students since students typically found in community college elementary algebra and intermediate algebra are in the same class. Therefore, you need to find ways to level the playing field so that some students are not bored and others lost. Of course, this level playing field means that all students must be able to bring something to the task that puts them into play, so to speak. Second, the authors build some of these skills directly into the lesson. For example, students plot points in Task 2, but the plotting of points is connected to constructing a scatterplot in which no association exists between the variables. Point plotting is in the service of the learning goal of interpreting trends in bivariate data. Third, research suggests that students need to struggle with meaningful tasks before a careful explication of concepts helps them construct deeply held understandings. The lesson is constructed to provide an opportunity for productive struggle, followed by carefully constructed tasks that the authors think are sufficient for all students to begin to make sense of the concepts at hand.

If you anticipate that some students, due to deficits in developmental math skills, may have trouble at specific points in the lesson, observe students work to see if your expectations are founded. Remember, the authors' hypothesis that *productive* struggle is a key ingredient in facilitating conceptual learning. You will need to use your own judgment about when to intercede to work with students individually. Report your observations back to the Lesson Study group. This will be invaluable information for improving the lesson.

Lesson Structure

This lesson has the following components:

- Introduction to the context of the lesson (15–20 minutes)
- Part I: Students work on a rich task, wrap-up, and transition to Part II (20–25 minutes)
- Part II: Scaffolded conceptual tasks (via discussion/group work/lecture) and wrap-up (40–50 minutes)
- Part III: Homework (done outside of class) (5 minutes)

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Introduction to the Context of the Lesson

(**Note:** To orient students to the data set, distribute the following handout and go over the introductory material before giving them a few minutes to choose the most nutritious cereal.)

Student Handout

In the United States, the Food and Drug Administration requires that most prepared food be labeled with nutrition information. Today part of the data set for the lesson was gathered from the list of required nutrition facts that appear on boxes of breakfast cereals.

Below are the nutrition facts for three cereals. There is a lot of information given. An entire math lesson could probably be designed around reading and understanding the information given on cereal labels. For now though, you will look over the labels fairly quickly, like you might do in the grocery store. Use your initial impressions to pick the cereal that you would rate as the most nutritious (the best for your health). You have a few minutes to do this.

Nutrition Facts
Serving Size 1.0 cup (1 NLEA serving)

Amount Per Serving	
Calories 117	Calories from Fat 9
	% Daily Value *
Total Fat 1g	1%
Saturated Fat 0g	1%
Polyunsaturated Fat 0g	
Monounsaturated Fat 1g	
Cholesterol 0mg	0%
Sodium 171mg	7%
Total Carbohydrate 26g	9%
Dietary Fiber 1g	3%
Sugars 14g	
Protein 1g	

Vitamin A 0% • Vitamin C 10%
Calcium 10% • Iron 30%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily value may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Nutrition Facts
Serving Size 1.0 cup (1 NLEA serving)

Amount Per Serving	
Calories 101	Calories from Fat 1
	% Daily Value *
Total Fat 0g	0%
Saturated Fat 0g	0%
Polyunsaturated Fat 0g	
Monounsaturated Fat 0g	
Cholesterol 0mg	0%
Sodium 202mg	8%
Total Carbohydrate 24g	8%
Dietary Fiber 1g	3%
Sugars 3g	
Protein 2g	

Vitamin A 0% • Vitamin C 10%
Calcium 0% • Iron 54%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily value may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Nutrition Facts
Serving Size 1.0 cup (1 NLEA serving)

Amount Per Serving	
Calories 111	Calories from Fat 16
	% Daily Value *
Total Fat 2g	3%
Saturated Fat 0g	2%
Polyunsaturated Fat 1g	
Monounsaturated Fat 1g	
Cholesterol 0mg	0%
Sodium 213mg	9%
Total Carbohydrate 22g	7%
Dietary Fiber 4g	14%
Sugars 1g	
Protein 4g	

Vitamin A 0% • Vitamin C 10%
Calcium 12% • Iron 69%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily value may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

(**Note:** These cereals are [from left to right] General Mills Cocoa Puffs, Kellogg's Corn Flakes, and General Mills Cheerios. Do not tell students the name of the cereals until after they have made their choices because the names might divert their attention from using the data to make their decision.)

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Discussion

After giving students a few minutes to make a decision, take a quick tally to see which cereal students chose as most nutritious. Then call on a few students to describe which ingredients they focused on. The goal in this short interaction with the class is to highlight the following three points to get students thinking about how ingredient amounts might be associated with nutritional ratings:

- Different people might devise different ways to rate the nutrition of cereals by focusing on different ingredients.
- You could use one ingredient or more than one ingredient to determine a nutrition rating.
- In a rating system, there is an association between the ingredient and the nutrition rating. For example, large amounts of sugar give lower ratings. Large amounts of fiber might give higher ratings.

Now transition to the *Consumer Reports* ratings by using the following information on the student handout to discuss *Consumer Reports*.

Part I [Student Handout]

Instead of making up your own rating system, you are going to investigate the *Consumer Reports* nutritional ratings for 77 breakfast cereals. *Consumer Reports* is published by a nonprofit organization called the Consumers Union, whose mission it is to work for a fair, just, and safe marketplace for all consumers and to empower consumers to protect themselves. *Consumer Reports* rates products based on its own criteria and testing. It prides itself on producing objective results. *Consumer Reports* maintain its objectivity by not allowing advertising within their publications and not allowing use of their results for commercial gain. (Retrieved from www.consumerreports.org/cro/aboutus/mission/overview/index.htm)

Consumer Reports uses a rating system with a scale of 0 to 100. Here is the distribution of *Consumer Reports* ratings for 77 cereals:

- What does each dot represent in this distribution?
- For this distribution, what seems to be an average rating?
- How would you describe the variability in ratings?
- How would you describe the shape of this distribution? What does the shape suggest about the rating system?

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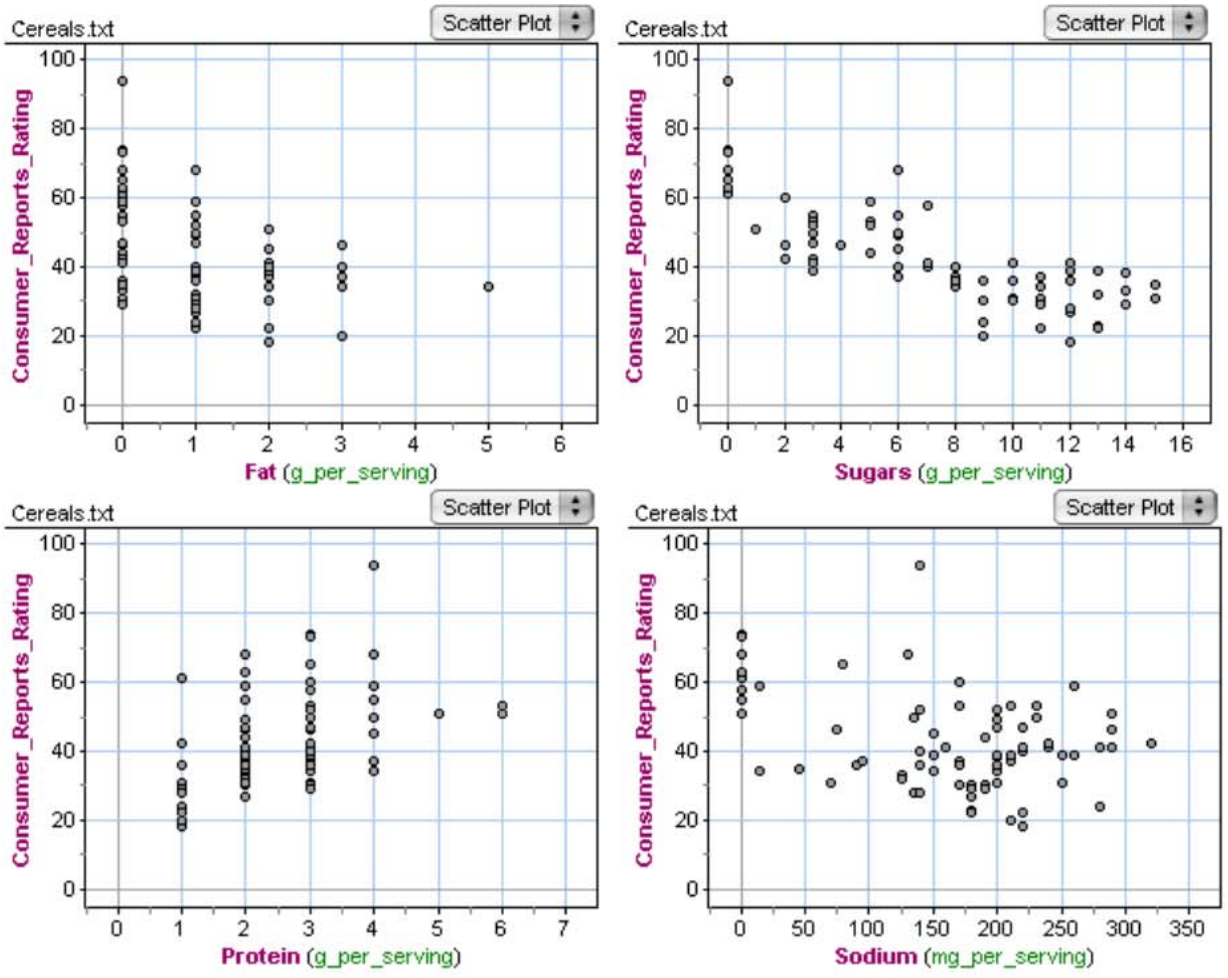
(**Note:** Students should be able to describe shape, center, and spread of this distribution based on what they have learned in Module 2. Answers can vary. Reasonable responses include the following: The dots in the distribution are cereals; The average rating is about 40 [actual mean is 44.0, median 41.0]; Ratings fall between about 18 and 94 on a scale of 0 to 100, with a reasonable estimate for the standard deviation being 10 rating points [actual standard deviation is 13.8, Q1 34, Q3 52.9].)

What you cannot tell from the dotplot is how the cereal ingredients (such as sugar or fat) are related to the ratings. You need a new type of graph, called a scatterplot, to investigate how two variables relate to each other. The scatterplots below show the amount of an ingredient in a serving of cereal and the *Consumer Reports* rating for 77 breakfast cereals.



The *Consumer Reports* rating formula is not made public. So, you do not know which ingredients are used in its rating formula. In this lesson, you will try to "crack their code" in a sense. Use the data to figure out which ingredients *Consumer Reports* may, or may not, use in their rating formula. The only clues you have are these scatterplots.

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The first step in this investigation is to answer the following two questions AND write down enough of your reasoning that someone can follow your thinking.

Two new cereals are being rated by Consumer Reports*. Cereal A has 10.5 grams of sugar in a serving and Cereal B has 2.5 grams of protein in a serving.*

(1) Based on the data shown, predict the *Consumer Reports* rating for the two cereals.

(2) For which cereal do you think your prediction is probably more accurate (more likely to be closer to the actual Consumer Report rating)? Why?

Note to Instructors

Give students about 3 minutes to work on these questions alone and then in small groups for 5–10 minutes (depending on your sense of whether productive conversations are occurring).

Part I provides students the opportunity to struggle with important ideas (like interpreting scatterplots and seeing patterns that relate to a question at hand). So, at this point in the lesson, you do not need to guide students to discover canonical ideas, such as correlation, or even correct their misconceptions or fix their errors. This is an informal introduction to distributional thinking with bivariate data. While students work, listen to how students are reasoning as they discuss the task. In the wrap-up, you have the opportunity to talk with students in general terms about making predictions and using a visual sense of the variability in the data to determine which ingredient is a more accurate predictor of ratings. In the wrap-up, you can refer to what you observed as students worked, giving praise and noting interesting aspects of their conversations that are relevant to learning goals for the lesson.

The next segment of the lesson, Part II, is designed with more explicit attention to developing the skills and understandings described in the learning outcomes. It is in Part II that you make connections as well correct errors and address misconceptions as appropriate.

Wrap-Up/Direction Instruction About Statistical Concepts

You will need to project the scatterplots for this discussion. To see if students are using the patterns in the data to make predictions, ask them to determine if the following predictions for ratings are reasonable or unreasonable (perhaps ask students to show a thumbs up for *reasonable prediction* and thumbs down for *unreasonable prediction*): Cereal A: 10, 30, 60; Cereal B: 10, 30, 60.

Plot each of these predictions on the scatterplot and highlight how the prediction fits the pattern in the data or deviates from the pattern.

Cereal A: 10 (not reasonable), 30 (reasonable), 60 (not reasonable); Cereal B: 10 (not reasonable), 30 (reasonable), 60 (reasonable).

Discuss the following questions through a brief minilecture or class discussion:

- What is a range of reasonable predictions for ratings of Cereal A? of Cereal B?
- Which ingredient, sugar or protein, is a more accurate predictor of *Consumer Reports* ratings?

Reasonable Answers: For Cereal A, there is a narrower range of reasonable predictions for the *Consumer Reports* rating (approximately 20 to 40), so the amount of sugar is a fairly accurate predictor of the rating. Draw a vertical line segment at $x = 10.5$, $20 < y < 40$ to visually represent this reasonable range of predictions. For Cereal B, there is a wider range of reasonable predictions for the *Consumer Reports* rating (approximately 25 to 70), so the amount of protein is not as accurate a predictor of the rating. Represent this range with a vertical line segment. There are other factors influencing the rating besides the amount of protein. So, it is more likely that your prediction of the rating for Cereal A is closer to the actual rating than your prediction for Cereal B.

Part II: Scaffolded Conceptual Tasks

Task 1: Reading and Interpreting Scatterplots

Introduction *[Student Handout]*

In this task, you are going to take a short detour from your investigation into which ingredients are the best predictors of *Consumer Reports* ratings. Here, you will work on interpreting scatterplots just to make sure everyone is comfortable with reading this new type of graph.

Activities *[Student Handout]*

(3) Captain Crunch has the lowest *Consumer Reports* rating of the 77 cereals in the data set. How much fat is in a serving of Captain Crunch?

(4) In this set of 77 cereals, Product 19 has the most sodium in a serving. What is the rating for Product 19?

(5) All-Bran Extra Fiber is the cereal with the highest rating. How much sugar, fat, and sodium are in a serving of All-Bran Extra Fiber?

Wrap-Up/Direct Instruction About Statistical Concepts

If students have been working in groups, you will have some sense of where they had difficulty. For the subsequent whole-class discussion, address areas of difficulty and answer questions in the context of the wrap-up questions. You will not have time to go over the answers to the scaffolded questions.

To bring closure, ask students to discuss the following questions or deliver a short minilecture that answers these questions:

> When a statistician reads a scatterplot, he or she asks herself two questions: (1) Who or what is described by the data? (i.e., What does a dot represent?) and (2) What measurements were made? (i.e., What are the variables?) Pick a scatterplot, and answer these two questions.

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Conceptual Task 2: Seeing Patterns and Relationships in Scatterplots

Introduction *[Student Handout]*

Now you will continue your detective work with *Consumer Reports* ratings. Try to identify ingredients that are good predictors of ratings and ingredients that are not. More importantly, focus on how patterns in the data are related to identifying ingredients that are good predictors.

(**Note:** If students are working in groups, for most of these activities, the authors suggest that you intervene if students have answers that are wildly off base. The goal is to foster distributional thinking with bivariate data. So, when you intervene, refrain from correcting; instead get students to talk about what they are seeing. Make observations or ask questions to nudge them in the right direction. Remember that in the wrap-up, you will provide direct instruction relative to the learning goals for the lesson, so you do not have to fix everything during group work.)

Activities *[Student Handout]*

(6) There are four cereals that have 3 grams of fat in a serving. Estimate the ratings for these four cereals. What might explain the variability in the ratings?

(**Answers:** Estimates might vary: 20, 34, 37, 46. Intervene and correct if students are misreading the scatterplots. Variability is explained by the impact of other ingredients on the ratings.)

(7) Imagine changing the recipe for a cereal that has 0 grams of fat in a serving and a rating of 60. Increase the amount of fat to 3 grams in a serving. Do you think the rating will probably increase or decrease or remain about the same? Or do you think that it is impossible to use the scatterplot to predict the impact of this change on the rating? How does the pattern in the data support your decision?

(**Answers:** The rating will probably decrease. You see a downward trend in the data that suggests that larger amounts of fat in a serving tend to be associated with lower ratings. If students think that it is impossible to predict how the rating will change, ask them to say more about why this is so. The goal is to get students to begin to develop distributional thinking for bivariate data, so nudge them to think about trends and patterns. It is reasonable to be unsure about how the rating will change because of the noise in the data. One pattern they could describe to support the "cannot tell" response is the large amount of variability in ratings for cereals with the same amount of fat.)

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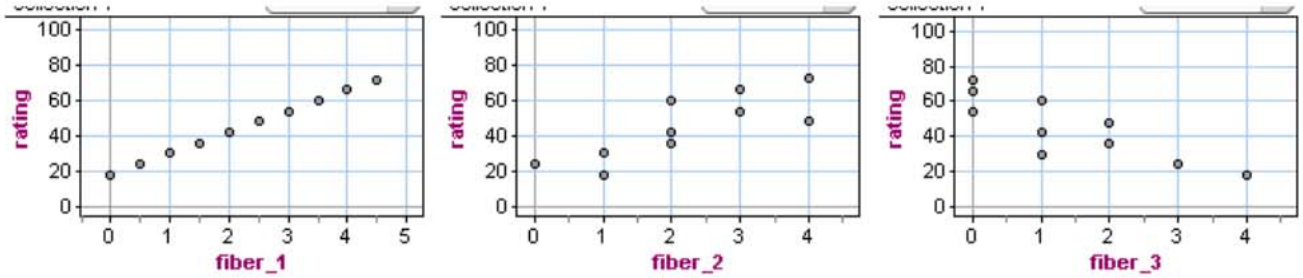
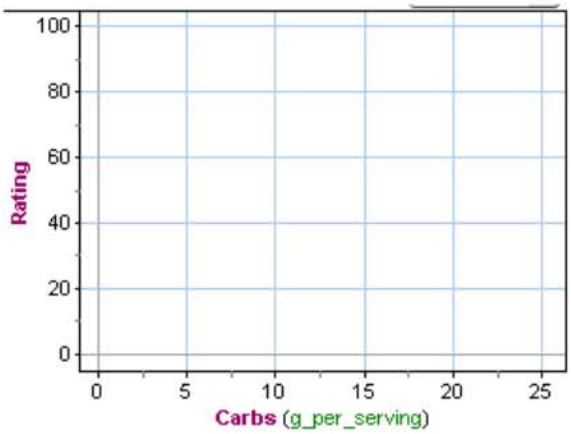
(8) Think about how the amount of fiber in a cereal might relate to the *Consumer Reports* rating. Here are three scatterplots with make-believe data from 10 made-up cereals. Which scatterplot do you think displays a pattern similar to what you may see in the actual data? Why?



(**Answer:** The middle graph. Intervene, if students pick another graph. Get them to talk about what they are seeing. Do they know that fiber is a good thing? Why would you expect some variability in ratings for cereals with the same amount of fiber?)

(9) Suppose that carbohydrates are not used in the *Consumer Reports* rating formula. Sketch a scatterplot with make-believe data from 10 make-believe cereals to illustrate what the data might look this situation.



(**Answer:** There are many possibilities here. If students do not have 10 dots in their graph, ask them why. Are they thinking that more than one cereal has the same measurements, so that dots are on top of each other? Or is there some confusion about what a dot represents or about the instructions. Also intervene if there is a strong association between the variables shown in their graph. With a strong association you might point out that they can predict the ratings with some confidence using their graph.)

Wrap-Up/Direct Instruction About Statistical Concepts

In this wrap-up, return to the issue of cracking the code on the *Consumer Reports* ratings by determining which ingredients appear to be good predictors of ratings. The real goal is to focus on issues related to building distributional thinking in this new setting of bivariate data. By the end of this wrap-up, the following should be clear to students:

- Upward and downward trends can help them make predictions even with very noisy data.
- The more scatter (variability) in the data, the less accurate their predictions probably are.

- When there is a lot of variability in the ratings for a fixed amount of ingredient, the ingredient is not a good predictor of the rating. Other factors are affecting the rating.

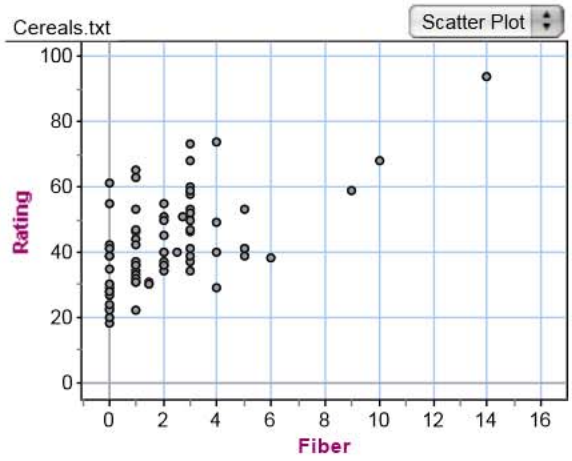
Compare and contrast fat-ratings and sugar-ratings scatterplots to address issues related to the previous activities:

Both scatterplots show a downward trend that indicates that cereals with smaller amounts of fat (or sugar) tend to have higher ratings and cereals with larger amounts of fat (or sugar) tend to have lower ratings. Draw a summarizing line with a negative slope or a highlight a downward diagonal region to highlight the trend. The downward trends indicate that increasing the amount of fat (or sugar) in a serving will tend to decrease the *Consumer Reports* rating.

The downward trends suggest that both fat and sugar are used in the *Consumer Reports* rating, but sugar is a better predictor of the ratings than fat because there is less variability in the ratings for similar cereals. Here you are reasoning in the same way that you did previously. For example, when you look at cereals with 1 gram of fat in a serving, you see a wide range of ratings falling between 20 and 70. You tend to see a similar wide range of ratings when you compare cereals with the same amount of fat. If you compare cereals with the same amount of sugar, you usually see less variability in their ratings. For example, cereals with 6 grams of sugar in a serving have ratings that differ, but the range of variability is less (about 35 to 65). The variability that we have been discussing is due to the impact of other ingredients on the rating.

Discussion of the Fiber-Ratings Relationship

Poll student responses to fiber-ratings problem (Question 8). Look at actual fiber data and compare and contrast it with the hypothetical data pattern that the majority chose.



In what ways does the scatterplot of the real data look like what they expected? You might have expected the upward trend in the data since smaller amounts of fiber would probably result in lower ratings and larger amounts of fiber in higher ratings. Highlight this upward trend with a line or a diagonal region.

In what ways is this graph surprising?
Again, you see a surprising amount of variability in the ratings for cereals with the same amount of fiber. This suggests that fiber is used in the rating formula but does not have as strong an impact as sugar. You might also be surprised that there are only a few cereals with more than 6 grams of fiber in a serving. These cereals have pretty high ratings.

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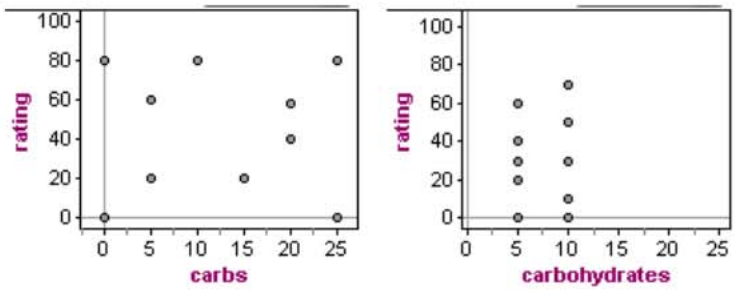
Discussion of Graphs Where You See No Relationship Between Carbohydrates and Ratings

Here are two graphs that illustrate no relationship between carbohydrates and ratings. In the graph on the left, there is no discernable pattern that can be used to make predictions. In the graph on the right, there is a vertical stripe pattern, but knowing the amount of carbohydrate does not help predict the rating.



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Part III: Homework [Student Handout]

(10) Summarize what you feel you learned today.

(11) The average *Consumer Reports* rating for these 77 cereals is 44. What is the largest amount of sugar per serving in a cereal that has above average ratings?

(**Answer:** 7 grams per serving.)

(12) Which is a better predictor of the Consumer Report ratings: sugar or sodium? Explain how the scatterplots support your answer.

(**Answer:** Sugar is a better predictor of ratings. There is a clear downward trend that makes it easier to predict the ratings based on a given amount of sugar. The sodium-ratings scatterplot has a lot of variability in ratings for cereals with similar amounts of sodium.)

(13) A friend says that she only pays attention to sugar amounts, even though she is also concerned by fat. Her reasoning is that low levels of sugar signal that the food also has low amounts of fat. Similarly, high levels of sugar signal that the food also has high amounts of fat. Does this appear to be true for breakfast cereals? Explain how the scatterplot supports your answer.

(**Answer:** This is not true for the breakfast cereals in this data set. The pattern described is an upward trend, but this graph does not have an upward trend. In this graph the cereals with highest amounts of sugar [14–16 grams in a serving], have low amounts of fat, 0–1 grams in a serving.)