MATH 1314 College Algebra
Day 11
Sections 4.4 and 4.5
Real Zeros of Polynomial Functions
The Fundamental Theorem of Algebra

Polynomial modeling Problem:

hours past midnight	Temperature in degrees Fahrenheit
0	0
1	3
2	0
3	-3
4	0

Describe the data in the table below with a third degree polynomial function.
Let $x = 0$ correspond to midnight.

The Factor Theorem

***x*-intercepts, zeros and factors**

Zeros with Multiplicity

Example:

Graphs and Multiplicity

The factorization of a polynomial function is unique.

Example: Write the complete factored form of the following polynomials with the given zeros.

(a) $f(x) = 7x^2 - \frac{63}{2}x + 14, \quad \text{zeros } \frac{1}{2},\ 4$

(b) $f(x) = 5x^3 - 5x^2 - 20x + 20, \quad \text{zeros } -2,\ 1, \text{ and } 2$

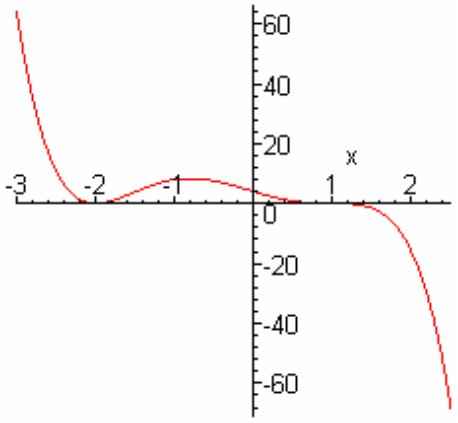
Using a graph to factor a polynomial function

Example: Use the graph of $f(x) = -\frac{1}{2}x^3 + x^2 + \frac{1}{2}x - 1$ to write the factored form of f.

Factoring symbolically

Example: Suppose 1 is a zero of $f(x) = -2x^3 - 6x^2 + 2x + 6$. Find the factored form of f.

Example: Write the complete factored form for the 5th degree polynomial shown below:



Rational Zeros Test:

Example: Use synthetic division along with factoring to find all rational zeros of $f(x) = -10x^3 - 17x^2 + 7x + 2$, then write f in complete factored form.

Descartes' Rule of Sign

Example: Determine the number of *possible* positive and negative real zeros of the polynomial function $f(x) = 3x^4 + 2x^3 - 8x^2 - 10x - 1$.

Section 4.5

The Fundamental Theorem of Algebra

A polynomial of degree *n* has at most *n* distinct real zeros

Example: Suppose all of the zeros for the given polynomials are distinct. Use their graphs to determine the number of real zeros and the number of imaginary zeros.

(a) $f(x) = -x^3 + 2x^2 + 2$ **(b)** $g(x) = -2x^2 + 2x - 3$ **(c)** $h(x) = x^4 - 4x^2 + 5$

Conjugate Zeros Theorem (for imaginary zeros and irrational zeros)

Example: Find all of the zeros of $f(x) = x^4 + x^3 + 2x^2 + x + 1$, given that one zero is $-i$.

Equation Solving:

Example: Find all solutions of the following equations

(a) $x^3 = 2x^2 - 5x + 10$ **(b)** $x^4 + x^3 = -x^2$