College Algebra

Sections 4.6 and 4.7

Rational Functions, Equations and Inequalities, Inverse Variation

Definition: A rational function

Domain:

Example: Determine the domain of the following rational functions

(a) $f(x) = \dfrac{x-2}{x^2-9}$ **(b)** $g(x) = \dfrac{x^2-2x-3}{2x^2-3x-14}$

Intercepts:

Asymptotes:

Vertical:

Notation:

Horizontal:

Notation:

Examples: Determine any vertical or horizontal asymptotes for the following rational functions

(a) $f(x)=\dfrac{x+3}{2x-1}$ **(b)** $f(x)=\dfrac{3x}{x^2-3x+2}$ **(c)** $f(x)=\dfrac{x^2-4}{x-2}$

Slant Asymptotes or Oblique Asymptotes

Example: Find all asymptotes and sketch a graph of $f(x)=\dfrac{2x^2-5x-2}{x-2}$

Using Transformations to graph a rational function

Basic graphs: $y=\dfrac{1}{x}$ $y=\dfrac{1}{x^2}$

Example: Use transformations to sketch a graph of the following:

(a) $f(x)=\dfrac{1}{x+1}-2$ **(b)** $f(x)=-\dfrac{1}{x^2}+3$

Graphing Rational Functions by Hand

Holes in a graph

A horizontal asymptote may be intersected!

Examples: p 321 # 88, 92

Section 4.7

Rational Equations, Inequalities, Inverse Variation

Example: Solve the following rational equations

(a) $\frac{6}{x^2} - 3 = \frac{3}{x}$ **(b)** $\frac{1}{x+4} + \frac{1}{x-4} = \frac{8}{x^2-16}$

Inverse Variation

Definition:

Example: The intensity of light *I* on an object is inversely proportional to the square of the distance of the object from the source of the light. At a distance of 3 meters, a 100-watt bulb produces an intensity of 0.88 watts per square meter. Find the constant of proportionality *k* and determine the intensity at a distance of 2 meters.

Polynomial and Rational Inequalities

Example: Solve for *x*, writing solution in interval notation

(a) $\frac{1}{x+2} \geq \frac{2}{x}$ **(b)** $3x^2 \leq x^3 + 2x$

Caution: multiplying both sides of an inequality by a variable, like *x* is not recommended. It can lead to incomplete or false solutions.

Students try p337 # 47, 68 and p 339 # 108