Definition: A rational function

Domain:

Example: Determine the domain of the following rational functions

(a) \( f(x) = \frac{x-2}{x^2-9} \)  
(b) \( g(x) = \frac{x^2-2x-3}{2x^2-3x-14} \)

Intercepts:

Asymptotes:

Vertical:

Notation:

Horizontal:

Notation:

Examples: Determine any vertical or horizontal asymptotes for the following rational functions
\( f(x) = \frac{x+3}{2x-1} \) \( f(x) = \frac{3x}{x^2 - 3x + 2} \) \( f(x) = \frac{x^2 - 4}{x - 2} \)

Slant Asymptotes or Oblique Asymptotes

Example: Find all asymptotes and sketch a graph of \( f(x) = \frac{2x^3 - 5x - 2}{x - 2} \)

Using Transformations to graph a rational function

Basic graphs: \( y = \frac{1}{x} \) \( y = \frac{1}{x^2} \)

Example: Use transformations to sketch a graph of the following:

\( f(x) = \frac{1}{x+1} - 2 \) \( f(x) = -\frac{1}{x^2} + 3 \)

Graphing Rational Functions by Hand

Holes in a graph

A horizontal asymptote may be intersected!

Examples: p 321 # 88, 92

Section 4.7
Rational Equations, Inequalities, Inverse Variation

Example: Solve the following rational equations

(a) \( \frac{6}{x^2} - 3 = \frac{3}{x} \)  
(b) \( \frac{1}{x+4} + \frac{1}{x-4} = \frac{8}{x^2-16} \)

Inverse Variation

Definition:

Example: The intensity of light \( I \) on an object is inversely proportional to the square of the distance of the object from the source of the light. At a distance of 3 meters, a 100-watt bulb produces an intensity of 0.88 watts per square meter. Find the constant of proportionality \( k \) and determine the intensity at a distance of 2 meters.

Polynomial and Rational Inequalities

Example: Solve for \( x \), writing solution in interval notation

(a) \( \frac{1}{x+2} \geq \frac{2}{x} \)  
(b) \( 3x^2 \leq x^3 + 2x \)
Caution: multiplying both sides of an inequality by a variable, like $x$ is not recommended. It can lead to incomplete or false solutions.

Students try p337 # 47, 68 and p 339 # 108