Sections 5.1 and 5.2

Combining Functions

Operations

Addition, Subtraction, Multiplication, Division, Composition

Domain

Example: Let $f(x) = x^2 + 1$ and $g(x) = \sqrt{x} - 1$.

Find the domain of f and g.

Find $(f+g)(x)$, $(f-g)(x)$, $(f \bullet g)(x)$, and $\left(\dfrac{f}{g}\right)(x)$ and the domain of each.

Evaluate: $(f+g)(4)$, $(f-g)(9)$, $(f \bullet g)(0)$, and $\left(\dfrac{f}{g}\right)(1)$, if defined.

Evaluating Combinations of Functions from a Graph.

Example: p 379 #31

Composition of Functions:

Illustration: There are 36 inches in a yard and 2.54 centimeters in an inch.

The function $f(x) = 36x$ converts *x* yards into inches. The function $g(x) = 2.54x$

Converts *x* inches into centimeters. To convert yards into centimeters, we combine

f and *g* in sequence. For example, convert 10 yd into centimeters.

This type of combining of functions is called composition and in this example it is the composition of *g* and *f* .

Pictorially:

Symbolically: $g\left(f(x)\right) = \left(g \circ f\right)(x)$

In our example: $g\left(f(x)\right) = g\left(36x\right) = 2.54(36x) = 91.44x$

And to convert 10 yards into centimeters
$g\left(f(10)\right) = g\left(36 \bullet 10\right) = 2.54(360) = 91.44 \bullet 10 = 91.44cm$

Note: This is not the product of the two functions:

Example: Suppose $f(x) = x^3 - 2$ and $g(x) = \sqrt[3]{x+2}$. Evaluate $g\left(f(2)\right)$ and $f\left(g(6)\right)$. How do they compare? Find the composite functions $g\left(f(x)\right)$ and $f\left(g(x)\right)$. Are they equivalent? Find the domain of each composition.

Example: Let $f(x) = \sqrt{x+2}$ and $g(x) = x^2$. Find the composite functions $g\left(f(x)\right)$ and $f\left(g(x)\right)$. Are they equivalent? Find the domain of each composition.

Find the following values, if defined: $g\left(f(2)\right)$, $g\left(f(-2)\right)$, $g\left(f(0)\right)$, and $f\left(g(-6)\right)$

Using a Graph to Evaluate a Composition

Example: p381 # 75

Evaluate a Composition from a table of values:

Example: p381 # 77

Decomposing a function:

Example: Find two functions *f* and *g* so that $h(x) = g\left(f(x)\right)$

(a) $h(x) = \left(x^3 - 1\right)^2$ **(b)** $h(x) = \dfrac{2}{x^2 - x + 1}$

Application: A weather balloon rises vertically at 2 feet per second from a point *P* on the ground. An observation camera is located on the ground 100 feet from *P*. Let *d* represent the vertical distance the balloon has risen and *s* the distance from the observation camera to the balloon at any time *t* in seconds.

(a) Write an equation expressing the vertical distance *d* as a function of time *t.*

(b) Write an equation expressing the distance *s* as a function of *d*

(c) Use composition to write an equation expressing the distance *s* as a function of *t.*

Section 5.2 Inverse Functions

Some examples of inverse operations

Example: State or write the inverse action or operation.

(a) Subtract 5 from x and divide the result by 3

(b) Opening the door and turning on the lights

Pictorial Representation of an inverse function and notation.

One-to-One Functions

Definition:

Note: Different inputs always result in different outputs

Determining if a function is one-to-one

Graphically

The Horizontal Line Test

Symbolically

Definition: Suppose a function f is one-to-one. Then the inverse of f exists and is the function f^{-1} where $f(a)=b$ implies $f^{-1}(b)=a$ for every a in the domain of f.

Properties of Inverse Functions

Cancellation Property

Domain and Range

Graph Symmetry

Finding and Verifying an Inverse Function

Example: Given the one-to-one function $f(x)=\sqrt[3]{x-4}$,

(a) Find a formula for $f^{-1}(x)$

(b) Identify the domain and range of $f^{-1}(x)$

(c) Verify your result with the cancellation property

Restricting the domain to find an inverse

Example: Let $f(x)=\left(x+2\right)^2$

(a) Does f have an inverse?

(b) Restrict the domain of f so that $f^{-1}(x)$ exists.

(c) Find $f^{-1}(x)$ for the restricted domain.

Numerical and Graphical Representations of Inverse Functions

Example: p 398 # 85, 89, 91, 103, 107, 109

Applications

Example: p 399 #121