College Algebra

Sections 5.3 and 5.4

Exponential and Logarithmic Functions

Exponential functions are used in numerous applications covering many fields of study. They are probably the most important group of functions in terms of their application. The main reason for this is because their rate of change is proportional to their amount at any time.

An amazing math fact

Form:

Domain

Range

Intercepts

Asymptotes

Graph $a > 0$ $0 < a < 1$ Exponential Growth or Decay

Linear Data vs Exponential

Example: Find a linear or exponential function that models the given data

(a)

x	-3	-2	-1	0	1
y	64	32	16	8	4

(b)

x	-2	-1	0	1	2
y	3	5.5	8	10.5	13

Application: Compound Interest

Formula: Derived from the simple interest formula $I = \Pr t$

Example: Suppose $15,000 is invested at an interest rate of 7% compounded annually. Calculate the account balance after 10 years. Now suppose the principal was compounded monthly. Calculate the account balance for the same time period.

The Natural Exponential Function – Continuous Compounding

Suppose $1 was invested into an account paying 100% annual interest. Fill in the table below where *n* is the number of compounding periods in one year and *A* is the amount in the account after 1 year.

n	$\left(1+\frac{1}{n}\right)^n$
1	
10	
100	
1000	
10000	
100000	
1000000	
10000000	
100000000	

This can be expressed in the following way:

$$\text{as } n \to \infty, \quad \left(1+\frac{1}{n}\right)^n \to e$$

Or in limit notation as:

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

From this limit, the continuous compounding of interest formula can be derived

$A = Pe^{rt}$

Where A is the amount at any time t,

P is the principal or initial deposit or amount invested

r is the annual rate of interest and

t is the time in years

Example: Suppose you invest $6,000 into a Roth IRA at age 22 and leave it in an account that earns 8% interest compounded continuously. At age 59 how much will you have in the account? If you leave the money in the account until age 75, how much will be in the account?

Natural Exponential Growth and Decay

Models:

Applications:

Example: A job offer at a recent college graduates job fair offers a starting salary of $44,000 with a guaranteed raise of 6% each year. Find a function f that computes the salary during the nth year.

Example: A bacteria culture initially contains 10,000 bacteria and is found to double in size every 4 hours. Find an exponential model of the form $f(x) = Ca^x$ that represents the amount of bacterial in the culture at any time *x* hours.

Example: Population Growth p 418 # 76

Example: Drug Concentration p 418 # 84

Example: Half Life p 419 # 89, 91

Note: the above problem can be modeled with a continuous decay model. In order to do this we need to be able to solve exponential equations of the form $e^x = b$, where *b* is a positive constant. We could approximate solutions using a calculator, however, there is a better way…

Section 5.4 The Logarithmic Function

A logarithmic function is best thought of as the inverse of a related exponential function.

For example, sketch a graph of the exponential function $y = 2^x$. Now, write an equation for the inverse of this function.

Definition: The logarithmic function with base *a*

Domain

***Inverse Properties**

Common Logarithm

Natural Logarithm

Evaluating Logarithms

Example: Evaluate the following:

(a) $\log_3 81$ **(b)** $\log_4\left(\frac{1}{64}\right)$ **(c)** $\ln e^{0.67}$

Graphs and Inverses

Sketch the following exponential functions with their inverse on the same set of axes. Write a logarithmic equation for each inverse.

(a) $f(x) = 2^x$, **(b)** $f(x) = \left(\frac{1}{2}\right)^x$, **(c)** $f(x) = 10^x$, and **(d)** $f(x) = e^x$.

Solving Exponential and Logarithmic Equations

Example: Solve each of the following exponential equations

(a) $2^x = \frac{1}{16}$ **(b)** $2e^x = 7$ **(c)** $5 \cdot 3^x - 6 = 24$

Using the inverse property, converting to logarithmic form, or equivalent base property

Example: Solve each of the following logarithmic equations

(a) $\log_6 x = -3$ **(b)** $\log_3 x = 4$ **(c)** $\ln x = 6.2$ **(d)** $6\log_2 3x = 12$

Converting to exponential form or using the inverse property

Change of Base Formula: $\log_a x = \frac{\log_b x}{\log_b a}$

Applications: p 436-437 # 129, 131, 134