College Algebra

Sections 5.5 and 5.6

Properties of Logarithms

More Exponential and Logarithmic Equations

Some essential properties:

For these properties, assume $m>0$, $n > 0$, $a > 0$, $a \neq 1$, and r any real number.

Example: Expand the following logarithmic expressions

(a) $\log_2 \frac{x}{y}$ **(b)** $\log \frac{\sqrt{x^3}}{k}$ **(c)** $\ln \frac{xy^3}{\sqrt{z}}$ **(d)** $\log_6 6y^3$

Example Write the following logarithms as a single expression

(a) $\ln 4e^2 + \ln 3e^{-2}$ **(b)** $6\log_5 x + \log_5 4x - \frac{3}{2}\log_5 z$

(c) $4\ln x - \frac{1}{3}\ln y + \frac{1}{2}\ln z$

Review Change of Base Formula

Example: Solve the following equation graphically

$\log_2\left(1-x^3-x\right)=e^x$

Students try p445 # 47, 55, 69, 85, 89

Section 5.6 More Exponential and Logarithmic Equations

Example: World population P in billions during the year x is modeled by $P(x)=3(1.017)^{x-1960}$, where $1960 \le x \le 2010$. Estimate the year when world population reached 4 billion.

Example: Light Absorption p 457 # 73

Example: Newton's Law of Cooling. If a hot object is put in a room with temperature T_0, then according to Newton's Law of Cooling, the temperature of the object after time t is modeled by $T(t)=T_0+Da^t$ where $0 < a < 1$ and D is the initial temperature difference between the object and the room. Suppose a professor brings a cup of tea initially $90°C$ into a room with a temperature of $25°C$, and it cools to $60°C$ after one hour. Find the values of $T_0, D, and\ a$ so that $T(t)=T_0+Da^t$ models the data. Find the temperature of the tea after 30 minutes. How long did it take for the tea to reach $50°C$? Graph the result.

Exponential Equations

Example: Solve the following exponential equations symbolically

(a) $9^{2x}=9^{4-x}$ **(b)** $6(1.3)^x-4=20$ **(c)** $4^{3x}=e^{x-2}$ **(d)** $\left(\frac{1}{2}\right)^{1-x}=\frac{1}{7}$

Logarithmic Equations

Example: Solve the following logarithmic equations symbolically

(a) $4\log_5 2x=6$ **(b)** $\log_2(4x-2)=3$

(c) $\log 20x=5-\log x$ **(d)** $\ln(x+1)=2\ln(x-1)$

Example: p 458 # 88, 97, 103

Note: Section 5.7 is not part of our course.