College Algebra
Day 18 Sections 6.3 and 6.4
Systems of Linear Equations in Three Variables
Solutions Using Matrices

The methods for solving a system of linear equations in three variables are similar to the methods for a system of linear equations in two variables.

Substitution, Elimination or a combination of both.

The nature of solutions is also similar to a system of linear equations in two variables.

No solutions, infinitely many solutions, exactly one solution.

The geometry is different. See page 524 for a geometric interpretation of a system of linear equations in three variables.

Example: Solve the following systems:

(a)
$$\begin{aligned} x + y - 2z &= 3 \\ -x - y + 3z &= -1 \\ x + 2y + z &= 12 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y - z &= 2 \\ 2x - y + z &= 2 \\ 5y - 3z &= 2 \end{aligned}$$

(c)
$$\begin{aligned} x + y + z &= 0 \\ x - y - z &= 3 \\ x + 3y + 3z &= 5 \end{aligned}$$

Application: Note: some applications in these sections may have no solution.

Example: p 517 # 31, 33, 35, 37

Section 6.4 Solutions Using Matrices

What is a matrix?

The **dimension** of a matrix

A **square** matrix

Representing a System of Equations:

The Coefficient Matrix **The Augmented Matrix**

Example: Express each linear system with an augmented matrix

(a) $$\begin{aligned} 3x - 2y &= 4 \\ -7x + 2y &= 1 \end{aligned}$$

(b) $$\begin{aligned} 2x + 3y - z &= 0 \\ 3y + z &= -1 \\ -x + 2y + z &= -4 \end{aligned}$$

Example: Convert the given augmented matrix back into the linear system it represents

(a) $$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 7 \\ 2 & 0 & 5 & -2 \\ 3 & -6 & 8 & -1 \end{array}\right]$$

(b) $$\left[\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -9 \end{array}\right]$$

Row-Echelon Form

A convenient form for solving systems of equations

main diagonal contains 1's or 0's.

first non-zero element in any no-zero row is 1 (called the leading 1)

If two rows contain a leading one, then the one with the left-most leading 1 is written first.

Rows containing only 0's occur last.

All elements below the main diagonal are zero.

Using back-substitution along with row-echelon form to solve a system

Example: Consider the following system:

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 0 & 1 & -4 & 30 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Solve using back-substitution.

Gaussian Elimination

Simply stated, this process uses the methods of elimination and substitution, using only the coefficients of the system, without the use of variables, sort of like synthetic division. It is most useful when the number of equations and variables increases.

Gaussian Elimination uses matrix row transformations to achieve row-echelon form or reduced row-echelon form. The row operations are:

1.
2.
3.

Example: Use Gaussian elimination with back-substitution to solve the following systems of equations.

(a)
$$\begin{aligned} 2x + 2y + z &= 2 \\ -x - y + 2z &= 4 \\ -2x + y + z &= -1 \end{aligned}$$

(b)
$$\begin{aligned} x + 3y + 4z &= 8 \\ -2x - y + 2z &= -1 \\ x + 8y + 14z &= 9 \end{aligned}$$

Reduced Row-Echelon Form

Every element above and below a leading 1 in a column is 0.

Example: The following matrices are in reduced row-echelon form. Solve the system

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Gauss-Jordan Elimination

Using matrix row transformations to transform an augmented matrix into reduced row-echelon form.

Example: Use Gauss-Jordan elimination to solve the following system.

(a) $\begin{aligned} x + y - z &= -2 \\ -x - 2y + z &= 2 \\ y + z &= 1 \end{aligned}$ **(b)** $\begin{aligned} x + z &= 2 \\ x - y - z &= 0 \\ -2x + y &= -2 \end{aligned}$

Applications:

Analytic Geometry: Determining a Quadratic Function
Example: p 534 # 83

Electricity
Example: p 533 # 77

Investments:
Example: p 533# 79