College Algebra
Day 19 Sections 6.5 and 6.6
Properties and Applications of Matrices
Inverses of Matrices

Matrix Notation

Notation used to denote elements in a matrix: a_{ij}

Two matrices are equal if
i) they have the same dimension
ii) corresponding elements are equal

Examples:

Operations with Matrices:

Addition/Subtraction

Only matrices with the same dimension can be combined. In either case the result is a matrix with the same dimension

Scalar Multiplication

Suppose k is a real number.

Example: Given $B = \begin{bmatrix} 4 & 0 & 6 \\ 1 & -1 & 0 \\ 2 & 9 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 1 & 20 \\ 1 & -3 & 0 \\ 0 & 10 & 3 \end{bmatrix}$,

Find the following:

(a) **B + C**. **(b)** **2B - 3C**.

Matrix Multiplication

The product of an *m* x *n* matrix *A* and a *n* x *p* matrix *B*, is the *m* x *p* matrix *AB*
To find the element a_{ij} in *AB*, multiply each element in the *i*th row of *A* by the corresponding element in the *j*th column of *B*. The sum of these products will be a_{ij}.

Example: Given $A = \begin{bmatrix} 0 & 5 \\ 4 & 0 \\ -1 & -2 \\ 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 & 6 \\ 1 & -1 & 0 \\ 2 & 9 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 & 20 \\ 1 & -3 & 0 \\ 0 & 10 & 3 \end{bmatrix}$,

$$D = \begin{bmatrix} -4 & 8 & -3 & 7 \\ 3 & 0 & -2 & 9 \end{bmatrix} \text{ and } E = \begin{bmatrix} -3 & 0 \\ 5 & 2 \end{bmatrix}$$

Find the following products, if possible.

(a) $A \cdot E$ **(b)** $B \cdot E$ **(c)** $B \cdot C$

Applications

Example: Tuition Costs p 548 # 67

Example: Car Sales p 548 # 70

Example: A manufacturer produces three models of a product, which are shipped to two warehouses. The number of units *i* that are shipped to warehouse *j* is represented by a_{ij} in matrix A below. The price per unit is represented by matrix B. Find the product BA and interpret the results.

$$A = \begin{bmatrix} 1000 & 3000 \\ 2000 & 4000 \\ 5000 & 8000 \end{bmatrix} \qquad B = \begin{bmatrix} \$25 & \$20 & \$32 \end{bmatrix}$$

Section 6.6 Inverses of Matrices

For an interesting understanding of matrix inverses, please read the computer graphics application which begins this section.

The Identity Matrix

The *n* x *n* **identity matrix**, denoted by I_n

Matrix Inverses:

Suppose A is an *n x n* matrix. If the inverse of *A*, denoted A^{-1} exists, then it is the *n x n* matrix satisfying $A \cdot A^{-1} = A^{-1} \cdot A = I_n$

If A^{-1} exists, then it is called invertible or nonsingular.
If *A* does not have an inverse, then it is singular

Example: Verify that A and B are inverses if

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Finding an inverse symbolically

Example: Find A^{-1} if $A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

Start by forming the 3 x 6 augmented matrix:

Next, use row operations to obtain I_3 on the left side of the augmented matrix

The right side is the inverse.

Check your work by showing that the right side matrix satisfies $A \cdot A^{-1} = A^{-1} \cdot A = I_n$

Example: Find A^{-1} for the 2 x 2 matrix $A = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}$

Solving Linear Systems with Inverses and Matrix Equations

A matrix equation can be used to represent a system of linear equations.
Consider the following system:

$$\begin{aligned} 2x + y - 3z &= 1 \\ y - 4z &= 6 \\ -3x + y + 5z &= 4 \end{aligned}$$

Let $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & -4 \\ -3 & 1 & 5 \end{bmatrix}$ be the coefficient matrix

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the variable matrix and $B = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$ be the constant matrix.

Find the product $A \cdot X$.

The matrix equation $A \cdot X = B$ is equivalent to the original system.
Any linear system can be represented by a matrix equation.

Solving for *X* in this matrix equation should give us the solution to the system.

Special note: Since matrix multiplication is not commutative, you the student must remember to multiply each side of the matrix equation *on the left* by A^{-1}.
In general $A^{-1} \cdot B \neq B \cdot A^{-1}$.

Example: Solve the following system using a matrix equation.

$$\begin{aligned} x - 3y &= 2 \\ 2x + 4y &= -6 \end{aligned}$$

Example: Solve the following system using a matrix equation.

$$\begin{aligned} x + y - z &= 0 \\ 2x - y - z &= 2 \\ -x + y + z &= 4 \end{aligned}$$

Applications

Example: p 561 # 71