College Algebra
Day 20 Sections 6.7
Determinants

Definition: The **determinant** of a 2 x 2 matrix is the single real number defined by

Example: Find the det C, if C is the matrix

$$C = \begin{bmatrix} -2 & 6 \\ -1 & 4 \end{bmatrix}$$

A square matrix A is invertible if and only if the det $A \neq 0$.

Is C from above invertible?

Minors and Cofactors:
In this course we will consider minors and cofactors for 3 x 3 matrices only.

Suppose we have a 3 x 3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The **minor** M_{ij} for element a_{ij} is the single real number
found in the following way:

The **cofactor**, denoted A_{ij} for the element a_{ij} is defined by

For example $M_{12} = \det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$ and $A_{12} = (-1)^{1+2} M_{12} =$

Example: Find M_{11} and A_{11} and M_{23} and A_{23}

Calculating the determinant of a 3 x 3 Matrix:

1. Select a row or column. This is called expanding about the _____.

2. Multiply each element in your selection by its cofactor

3. The sum of these products is the determinant

Find the det C, if C is the matrix $C = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 4 & -2 \\ 1 & 1 & 3 \end{bmatrix}$

The sign matrix

Applications:

Cramer's Rule The solution to a linear system of the form

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

is given by

Example: Use Cramer's Rule to solve the system $\begin{aligned} 2x + y &= -3 \\ -4x - 6y &= -7 \end{aligned}$

Area of Regions:

If a triangle has vertices $(a_1,b_1);(a_2,b_2);(a_3,b_3)$, then its area is equal to $|D|$,

where $D=\frac{1}{2}\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{bmatrix}$.

Equations of Lines: If a line passes through points (x_1,y_1) and (x_2,y_2) then an equation of the line can be found by calculating

$$\det\begin{bmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix}=0$$

Example: Find the standard form $ax+by=c$ of the line passing through $(6,-7)$ and $(4,-3)$