College Algebra

Day 8 Sections 3.4 and 3.5

Quadratic Inequalities and Transformations of Graphs

The quadratic function \( f(x) = \frac{1}{9}x^2 + \frac{11}{3}x \) is sometimes used to model the stopping distance for a car traveling at \( x \) miles per hour on a wet level pavement. Suppose a driver can only see 200 feet ahead, then safe driving speeds satisfy the quadratic inequality \( \frac{1}{9}x^2 + \frac{11}{3}x \leq 200 \). Solving this type of inequality is the topic of Section 3.4.

Graphical and Numerical Solution

Example: Find an equation for the quadratic function shown

1. (a) Use the graph of the quadratic function \( f \) to write it as \( f(x) = a(x - h)^2 + k \)
   (b) Use the graph to solve the inequality \( f(x) < 0 \)

Example: Solve the illustration about stopping distance
Symbolic Solutions to Quadratic Inequalities

Suppose you wish to solve a quadratic inequality $ax^2 + bx + c > 0$, where $>$ may be replaced with $\geq$, $<$, or $\leq$.

The process:

Example: Solve the following inequalities symbolically. Write the solution using interval notation.

(a) $x^2 \leq 15 - 2x$

(b) $x^2 + 4x - 3 > 0$

Application: p 220 # 65

Section 3.5 Transformations of Graphs
Translations:
Vertical
Horizontal

Transformations
Stretching-Vertically and Horizontally
Shrinking or Compressing-Vertically and Horizontally

Reflections
About the x-axis
About the y-axis

Let $f$ be a function and $c > 0$

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Effect on the graph of $f$</th>
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<tbody>
<tr>
<td>$f(x) + c$</td>
<td></td>
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<tr>
<td>$f(x) - c$</td>
<td></td>
</tr>
<tr>
<td>$f(x + c)$</td>
<td></td>
</tr>
<tr>
<td>$f(x - c)$</td>
<td></td>
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<tr>
<td>$c \cdot f(x)$, $c &gt; 1$</td>
<td></td>
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<tr>
<td>$c \cdot f(x)$, $0 &lt; c &lt; 1$</td>
<td></td>
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<tr>
<td>$f(cx)$, $c &gt; 1$</td>
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<tr>
<td>$f(cx)$, $0 &lt; c &lt; 1$</td>
<td></td>
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<tr>
<td>$-f(x)$</td>
<td></td>
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<tr>
<td>$f(-x)$</td>
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**Example:** Sketch the basic graph related to the following functions, then describe how the graph of each equation can be obtained by transforming the graph of the basic related graph. Make a sketch.

(a) \( y = |x - 2| + 3 \)

(b) \( y = -\frac{1}{2}(x + 3)^2 \)

(c) \( y = -\sqrt{1 - x} \)

**Numerical Transformations**

**Example:** Two functions \( f \) and \( g \) are related by the given equation. Use the numerical representation of \( f \) to make a numerical representation of \( g \).

\[ g(x) = f(-x) + 1 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>11</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
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Students Try p 237 #86

**Transforming Graphical Representations**
Given the graph of \( y = f(x) \) below, graph

(a) \(-f(x)\)
(b) \(\frac{1}{2} f(x)\)
(c) \(f(-x)\)
(d) \(y = 2f(x-1)\)