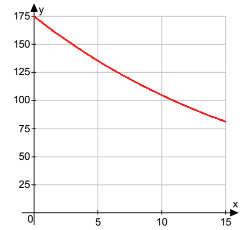
Chapter 5 Review Exercises Answers

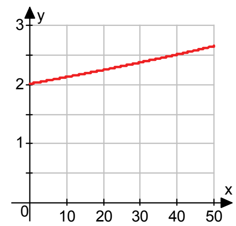
1. Write an exponential function that represents:

   a. Exponential decay of 5% with an initial value of 175. Fill in the table of values and sketch a graph.



x	0	5	10	15
$f(x)$	175	135.41	104.78	81.076

   b. Exponential growth of 0.66% with an initial value of 2.012. Fill in the table of values and sketch a graph.



x	0	10	20	30
$f(x)$	2.012	2.149	2.296	2.453

c.

2. For the following exponential functions, state whether it represents exponential growth or decay. Find the initial value, the base and the rate of growth or rate of decay.

   a. $P(t) = 0.0125(1.0625)^t$
   growth; initial 0.0125; base (1.0625); growth rate 6.25%

   b. $f(t) = 350(0.86)^t$
   decay; initial 350; base (0.86) decay rate 14%

3. An unpaid credit card balance will grow exponentially according to the model $B(t) = 3961(1.1998)^t$, where $B(t)$ is the unpaid balance (in dollars) and t is in years. Use the model to answer the following questions.

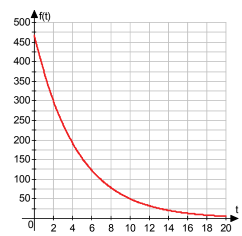
   a. What is the original balance? $3961
   b. What is the rate of growth? 19.98%
   c. What will the balance be in 5 years? $9848.02
   d. What will the balance be in 10 years? $24,484.62
   e. Suppose the balance is paid off after 15 years. How much interest will be paid? $59,913.82
   f. *T* Use the graph to estimate when the unpaid balance would be equal to $10,000. about 5 years

4. Suppose water that contains 468 ppm (parts per million) of impurities is forced through a filter that removes 20% of the impurities per foot of filter. Use this to answer the following questions.

   a. Write a function that gives the amount of impurities in the water as a function of the length of the filter and state the multiplier. $f(t) = 468(0.80)^t$; multiplier 0.08
   b. Create a table of values for even integers from 0 to 20 feet of filter and sketch a scatterplot.

t ft	0	2	4	6	8	10	12	14	16	18	20
f(t)	468	299.52	191.69	122.68	78.517	50.251	32.161	20.583	13.173	8.4307	5.3957

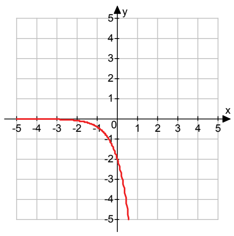
   c. Estimate the length of a filter that will eliminate 99% of the impurities. about 20.64 ft.



5. Graph the following exponential functions and identify the transformation or transformations used to obtain the graph from the graph of $f(x) = 5^x$. Identify the horizontal asymptote.

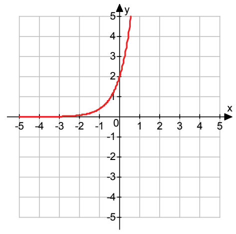
a. $g(x) = -2 \cdot 5^x$

reflect across x-axis, stretch by a factor of 2



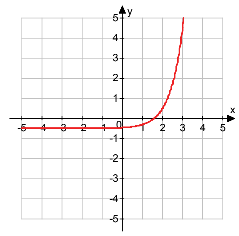
b. $g(x) = 2 \cdot 5^x$

stretch by a factor of 2



c. $g(x) = 5^{x-2} - \frac{1}{2}$

shift right 2 units and down ½ unit



6. Suppose \$12,500 is invested at 4.35% interest. Find the amount in the account at the end of 25 years if the interest is

a. simple interest
\$26,093.75

b. compounded annually
\$36,242.77

c. compounded monthly
\$37,012.74

d. compounded quarterly
\$36,868.51

e. compounded weekly
\$37,068.74

f. compounded daily
\$37,083.19

7. Simplify the following expressions. Write the answers using positive exponents.

a. $e^{-7x}e^{-3x}$

$e^{-10x} = \frac{1}{e^{10x}}$

b. $\frac{e^{-8x}}{e^{-11x}}$ e^{3x}

c. $\frac{e^{2x-1}}{e^{3-5x}}$ e^{7x-4}

d. $e^{-x}(e^x + 2e^{-x}) - 3e^{-2x}(e^x - 1)$ $1 - 3e^{-x} + 5e^{-2x}$

8. Find the amount in an account after 12 years if \$1525 principal is invested at 5% compounded continuously. \$2778.73

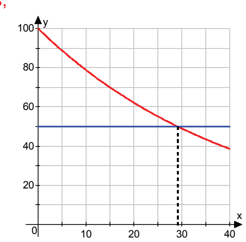
9. Strontium-90 (Sr_{90}) is found in waste from nuclear reactors and it is considered one of the more hazardous constituents of nuclear wastes. The amount of Sr_{90} remaining after t years is given by $A(t) = Pe^{-0.0238t}$. Suppose a 100 gram sample is used.

a. Write $A(t)$ the function that gives the amount of the 100 gram sample remaining. $A(t) = 100e^{-0.0238t}$

b. Find and interpret $A(10)$, $A(30)$.

$A(10) = 78.82$ grams left after 10 years; $A(30) = 48.97$ grams left after 30 years;

c. Graph the amount of strontium-90 remaining from the 100 gram sample after t years for $0 \le t \le 40$.



d. Use the graph to approximate the half life of strontium-90. 29 yrs.

e. Find the half life of strontium-90 symbolically. 29.12 yrs

10. Convert the following exponential equations to logarithmic form.

a. $e^4 = x$ $4 = \ln x$

b. $e^{-x} = \frac{1}{e^x}$ $\ln e^{-x} = \ln \frac{1}{e^x} = -\ln e^x$

c. $10^{-2} = \frac{1}{100}$ $\log \frac{1}{100} = -2$

Comment [VP1]: This is a basic exponent rule. What did you have in mind?

11. Write the following logarithmic equations in exponential form.

a. $\log \frac{1}{10} = -1$ $10^{-1} = \frac{1}{10}$

b. $x = \ln e^x$ $e^x = e^x$ *or* $x = x$

c. $\log 10000 = 4$ $10^4 = 10{,}000$

Comment [VP2]: I'm not sure what you had in mind for this one.

12. Find the value of x in each of the following equations.

a. $\log_2 x = 5$
$x = 2^5 = 32$

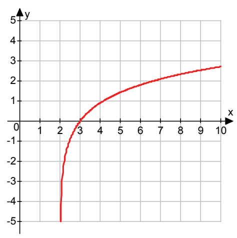
b. $\log_{16} x = \frac{1}{2}$
$x = 16^{1/2} = 4$

c. $\ln x = \ln 2$
$x = 2$

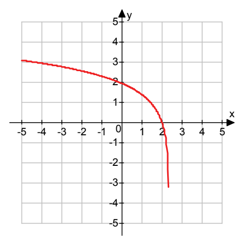
d. $\log x = 1$
$x = 10$

13. Find the domain of each of the following logarithmic functions, state the vertical asymptote and sketch the graph.

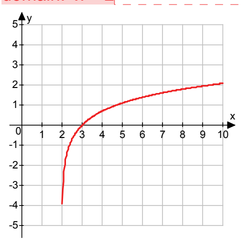
a. $f(x) = 3\log(x-2)$
domain: $x > 2$



b. $g(x) = \ln(-3x+7)$
domain: $x < \frac{7}{3}$



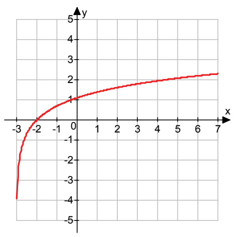
c. $h(x) = \ln(x-2)$
domain: $x > 2$



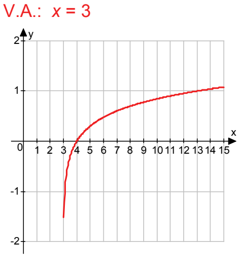
Comment [VP3]: This one may be too similar to part a.

14. Sketch the graph of the given function by using the graph of $y = \log x$ or $y = \ln x$ and transformations. Find the domain, the range, and the vertical asymptote of the function.

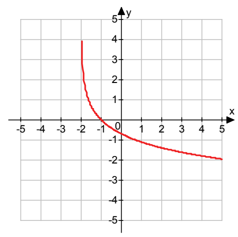
a. $f(x) = \ln(x+3)$
domain: $x > -3$;
range: $f(x) \in \mathbb{R}$
V.A.: $x = -3$



b. $f(x) = \log(x-3)$
domain: $x > 3$;
range: $f(x) \in \mathbb{R}$;
V.A.: $x = 3$



c. $f(x) = -\ln(x+2)$
domain: $x > -2$;
range: $f(x) \in \mathbb{R}$;
V.A.: $x = -2$



15. Express the following logarithmic expressions as the sum or difference of logarithms.

a. $\log_3\left[x(x+2)(x-2)\right]$ $\quad \log_3 x + \log_3(x+2) + \log_3(x-2)$

b. $\ln\left[(2x-3)^2(x+2)\right]$ $\quad 2\ln(2x-3) + \ln(x+2)$

c. $\log\sqrt[5]{\dfrac{x^3y^6}{z^3}}$ $\quad \dfrac{1}{5}\left[3\log x + 6\log y - 3\log z\right]$

d. $\ln\dfrac{(x-1)^3(x+3)^2}{(x+6)^5}$ $\quad 3\ln(x-1) + 2\ln(x+2) - 5\ln(x+6)$

16. Write each of the following as a single logarithm.

a. $\dfrac{2}{5}\ln x - \dfrac{1}{3}\ln y - \dfrac{7}{4}\ln z$ $\quad \ln\dfrac{x^{2/5}}{y^{1/3}z^{7/4}}$

b. $\dfrac{1}{3}\ln(x+2) - \dfrac{1}{2}\ln y - \dfrac{2}{5}\ln z$ $\quad \ln\dfrac{(x+2)^{1/3}}{y^{1/2}z^{2/5}}$

17. Solve the following exponential equations for x. Solve exactly and then round to three decimal places.

a. $5.1^x = 12.4$ $\quad x = \dfrac{\ln 12.4}{\ln 5.1} \approx 1.545$

b. $2^{x+1} = 3^{2x-3}$ $\quad x = 4$

c. $5^{x-2} = 6^{x-1}$ $\quad x = \dfrac{2\ln 5 - \ln 6}{\ln 5 - \ln 6} \approx -7.827$

d. $\left(\dfrac{9}{11}\right)^x = 2.125$ $\quad x = \dfrac{\ln 2.125}{\ln(9/11)} \approx -3.756$

18. Solve the following exponential equations for x. Solve exactly and then round to three decimal places where appropriate.

a. $10^{2x-1} = 0.15$ $\quad x = \dfrac{1+\log 0.15}{2} \approx 0.0880$

b. $e^{3x-1} = 5.57$ $\quad x = \dfrac{1+\ln 5.57}{3} \approx 0.906$

c. $10^{4x-5} + 2.1 = 11.25$ $\quad x = \dfrac{5+\log 9.15}{4} \approx 1.49$

d. $e^{2x-1} = 3.123$ $\quad x = \dfrac{1+\ln 3.123}{2} \approx 1.069$

19. Solve the following logarithmic equations.

Comment [VP4]: I think that we need more log equations, some with nicer solutions. I might also be good to include some where a single log expression is equal to a constant.

a. $\log\left(\dfrac{2x}{3x-2}\right) = \log 4$ $\quad x = \dfrac{4}{5}$

b. $\log(5x+1) = \log\left(x^2+3x-14\right)$ $\quad x = 5$; $x = -3$ is extraneous

c. $\ln\left(x^2-7\right) = \ln(2x+1)$ $\quad x = 4$; $x = -2$ is extraneous

d. $\log_2 x + \log_2(x+7) = 5$ $\quad x = \dfrac{-7+\sqrt{177}}{2}$

e. $\log(x+3) - \log(x-3) = 5$ $\quad x = \dfrac{100001}{33333} \approx 3.00006$

f. $\ln(3x+1) - \ln(2x-3) = 5\ln 2$ $\quad x = \dfrac{97}{61}$

20. Solve the following equations for the indicated letter.

a. $2P = P(1.07)^t$ for t. $\quad t = \dfrac{\ln 2}{\ln 1.07} \approx 10.245$

b. $10P = P(1.084)^t$ for t. $\quad t = \dfrac{\ln 10}{\ln 1.084} \approx 28.5475447$

c. $25(1.065)^t = 55(1.053)^t$ $\quad t = \dfrac{\ln\frac{25}{50}}{\ln\frac{1.053}{1.065}} \approx 61.17$

d. $A = 250e^{t/30}$ for t. $\quad x = 30\ln\dfrac{A}{250},\ A > 0$

21. Find f^{-1} for $f(x) = 2e^{-x}$. $\quad f^{-1}(x) = -\ln\dfrac{x}{2}$

22. Find f^{-1} for $f(x) = 3\ln e^{-x}$. $\quad f^{-1}(x) = -\dfrac{x}{3}$

Comment [VP5]: Is this the problem you intended? It reduces to y = -3x

23. How long does it take for money to double at 8% interest compounded annually? about 9 years

24. How long does it take for money to double at 11% interest compounded continuously? about 6.3 years

25. T The following table shows the average employee contribution for health care coverage for the given years.

Year	1998	1999	2000	2001	2002	2003
Cost	$627	$677	$740	$768	$972	$1196

Source: Hewitt Health Care Cost Analysis

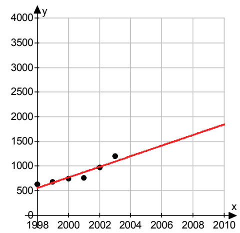
a. Find linear, quadratic, cubic and quartic models for the data.
b. Graph the models.
c. Estimate healthcare costs in 2009 using each model.

For a – c, x represents the year and y represents cost

linear:

a. $y = 107.37x - 213{,}966.5$

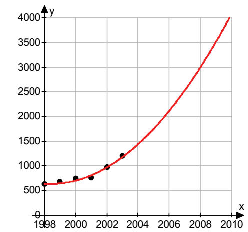
b.



c. In 2009: $1739.83

quadratic:

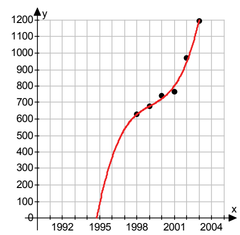
$y = 25.60714286x^2 - 102346.81x + 102265750$



c. In 2009: $3518.09

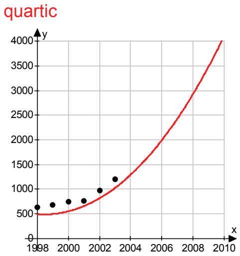
cubic

b. $y = 6.185x^3 - 37{,}094.78x^2 + 74{,}156{,}959.93x - 49{,}416{,}270{,}000$



c. $7051.06

quartic



$y_{TI} = -2.25x^4 + 18{,}010.68519x^3 - 54{,}064{,}082.89x^2 + 72{,}128{,}109{,}000x - 36{,}085{,}370{,}000{,}000$

$y_{AG} = 1.7283053 \times 10^{-6} x^4 + 0.1988393x^3 - 1209.2231827x^2 + 2.3955961 \times 10^6 x - 1.5726662 \times 10^9$

Comment [VP6]: I really hate how large the number are. Maybe we should delete the x = year thing and just go with x = years since 1990. I had a hard time getting a quartic graph to approximate the data. The first function listed is the regression using the TI and the function below it is from Advanced Grapher. The Advanced Grapher function gives the graph and the TI function doesn't show on the grid with the data.

d. Convert the data for employee contributions to healthcare coverage given in the table to years since 1998.

Year	1998	1999	2000	2001	2002	2003
Yrs since 1998	0	1	2	3	4	5
Cost	$627	$677	$740	$768	$972	$1196

Comment [VP7]: I think it would be better to let x represent years since 1990, not 1998.

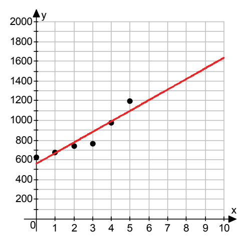
e. Find a linear, quadratic, cubic and quartic model for the data.
f. Estimate the healthcare costs in 2009 using each model.
g. Graph the cubic and quartic models on the same set of axes that extends to 2007.

For e – g, x represents years since 1998 and y represents cost

linear:

a. $y = 107.37x - 561.37$

b.

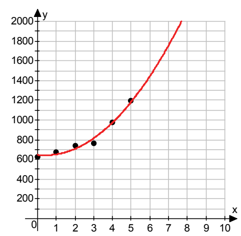


c. In 2009: $1742.66

quadratic:

a.. $y = 25.6x^2 - 20.66x + 646.93$

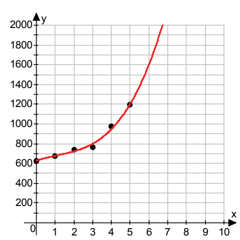
b.



c. in 2009: $3518.09

cubic

b. $y = 6.185x^3 - 20.78x^2 + 64.07x + 628.37$

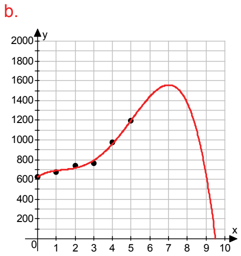


c. In 2009: $7051.06

quartic

a. $y = -2.25x^4 + 28.69x^3 - 89.89x^2 + 624.52$

b.



h. Describe the differences between the models as years increase. Which seems more reasonable?

The linear, quadratic, and cubic model predict that health care costs will increase. The linear model predicts that cost increases less rapidly than the quadratic mode, which in turn increases less rapidly than the cubic model. The quartic model predicts that health care cost will decrease after 2005 and become 0 mid 2007.